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$(\mathbb{Z}_2)^k$-ACTIONS WITH TRIVIAL NORMAL BUNDLE OF FIXED POINT SET

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1. Introduction

Let k be a positive integer. Let $\phi\colon (\mathbb{Z}_2)^k \times M^n \to M^n$ be a smooth action of the group $(\mathbb{Z}_2)^k = \{t_1, \dots, t_k \mid t_i^2 = 1 \text{ and } t_i t_j = t_j t_i\}$ on a closed smooth n-dimensional manifold M^n. It is well known that the fixed point set F of the action ϕ on M^n, i.e.,

$$F = \{m \in M^n \mid \phi(t, m) = m \text{ for all } t \in (\mathbb{Z}_2)^k\}$$

is a disjoint union $\bigsqcup_h F^{n-h}$ of closed submanifolds of M^n.

The purpose of this paper is to study $(\mathbb{Z}_2)^k$-actions having the property that each component of fixed point set has trivial normal bundle. When $k = 1$, Conner and Floyd gave the complete analysis of such actions (see [1, Theorem 25.1]). When $k > 1$, as far as the author knows, some works in this respect are only on the case in which the fixed point set consists of isolated fixed points. For example, see [1], [2], [3], [4], and [5].

In [5], a linear independence condition for the fixed point set of $(\mathbb{Z}_2)^k$-actions on closed manifolds was introduced, and then using the condition, one analyzed the property of fixed point set for $(\mathbb{Z}_2)^k$-actions having only isolated points. Following this idea, we first consider a more general case, i.e., $(\mathbb{Z}_2)^k$-actions with constant dimensional fixed point set satisfying that each component of fixed point set has trivial normal bundle. The result is stated as follows.

Theorem 1.1. *Suppose that (ϕ, M^n) is an $(\mathbb{Z}_2)^k$-action on a closed manifold M^n with constant l-dimensional fixed point set F^l for which each component of F^l has trivial normal bundle. Then either (ϕ, M^n) bounds equivariantly or the fixed point set has the following property*:
(1) *for $l < n$, F^l must possess the linear dependence property*;
(2) *for $l = n$, F^n is bordant to M^n.*

REMARK. In Theorem 1.1, linear dependence forces the fixed points of (ϕ, M^n) to have not only a normal representation, so F^l has at least two connected components

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