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INTEGRABLE IRREDUCIBLE HIGHEST WEIGHT MODULES FOR $sl_2(\mathbf{C}_p[x^{\pm 1}, y^{\pm 1}])$

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1. Introduction

Modules for loop algebras $g \otimes \mathbf{C}[t_1^{\pm 1}, \ldots, t_d^{\pm 1}]$ and their universal central extensions have been extensively investigated where g is a finite dimensional simple Lie algebra. The universal central extensions are affine Lie algebras in the case $d = 1$ and called toroidal Lie algebras in the case $d > 1$ [1]. Affine Lie algebras and their q-analogues have been well studied and found applications in several areas. Compared with these algebras, the investigations of the representation theory of toroidal Lie algebras are still under way, though much progress was made in references [1], [2], [3], [4], to name a few, and their q-analogues were applied in [5], [6], [7] and [8].

Among modules for these Lie algebras we are interested in integrable modules. In the case $d = 1$ (with the scaling element), in [9] and [10], irreducible integrable modules with finite dimensional weight spaces were classified and, when the central element acts trivially, they were shown to be isomorphic to loop modules of the tensor product of some modules depending on continuous parameters or their irreducible submodules. The q-analogue of this problem was investigated in [11]. In the case $d = 2$, integrable modules were studied in [4] where some of the central elements act nontrivially. For $d \geq 2$ these modules were considered in [3] and references therein.

In this paper we consider a loop algebra with the algebra of Laurent polynomials replaced by a quantum torus [12]. For generic $p \in \mathbf{C}^\times$ let $\mathbf{C}_p[x^{\pm 1}, y^{\pm 1}]$ be the $\mathbf{C}$ algebra of the Laurent polynomials in the two variables x, y satisfying $yx = pxy$. We shall denote this algebra by $\mathcal{C}_p$. Let $gl_2(\mathcal{C}_p)$ be the Lie algebra of 2×2 matrices with entries in $\mathcal{C}_p$ with the usual commutator. We consider the Lie algebra $sl_2(\mathcal{C}_p) := [gl_2(\mathcal{C}_p), gl_2(\mathcal{C}_p)]$. Lie algebras of this kind appeared in the study of some extended affin Lie algebras in [13] and when taking the $q = 1$ limit of the quantum toroidal algebras in [5]. Representations of these Lie algebras were considered in [14] and those of their central extensions were studied in [15], [16] and [17] in terms of vertex operators. The main result of this paper is the classification of integrable irreducible highest weight modules for $sl_2(\mathcal{C}_p)$. Our line of thought and result are similar to those in [9] and [10] but more complex.

This paper is organized as follows. In Section 2, after giving the necessary definitions, we state our result for the classification of integrable irreducible highest weight