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SOME RESULTS ON THE PHASE STRUCTURE OF THE TWO-DIMENSIONAL WIDOM-ROWLINSON MODEL

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1. Introduction

We study the phase structure of the two-dimensional (2D) lattice Widom-Rowlinson model. Let $\Omega = \{-1, 0, +1\}^{\mathbb{Z}^2}$ be the configuration space with product topology. The Borel σ-algebra of Ω is denoted by $\mathcal{F}$. For $\Lambda \subset \mathbb{Z}^2$, we consider $\Omega_\Lambda = \{-1, 0, +1\}^\Lambda$ and its Borel σ-algebra $\mathcal{F}_\Lambda$. We write $x \sim y$ if $x, y \in \mathbb{Z}^2$ are adjacent, namely $|x_1 - y_1| + |x_2 - y_2| = 1$. We say that x and y are $(*)$adjacent and write $x \overset{*}{\sim} y$ if $\max\{|x_1 - y_1|, |x_2 - y_2|\} = 1$. A configuration $\omega \in \Omega_\Lambda$ is said to be *feasible* if $\omega(x)\omega(y) \neq -1$ for all adjacent $x, y \in \Lambda$.

We write $\Lambda \Subset \mathbb{Z}^2$ if Λ is a finite subset of $\mathbb{Z}^2$. For $\Lambda \Subset \mathbb{Z}^2$ and a feasible boundary condition $\omega \in \Omega$, the *finite volume Gibbs distribution* $\mu^\omega_{\Lambda,\lambda,h}$ is defined by

$$\mu^\omega_{\Lambda,\lambda,h}(\sigma) = \frac{1}{Z^\omega_{\Lambda,\lambda,h}} 1_{\{\sigma*\omega:\ \text{feasible}\}} \prod_{x\in\Lambda} \lambda^{\sigma(x)^2} e^{h\sigma(x)}.$$

Here $\lambda > 0$ is a parameter called *activity,* and $h \in \mathbb{R}$ is a parameter which plays a similar role as the external field in the Ising model. The normalizing constant $Z^\omega_{\Lambda,\lambda,h}$ is called the *partition function*. The configuration $\sigma * \omega \in \Omega$ is defined by

$$\sigma * \omega(x) = \begin{cases} \sigma(x) & \text{if } x \in \Lambda, \\ \omega(x) & \text{if } x \in \Lambda^c. \end{cases}$$

A probability measure μ on $(\Omega, \mathcal{F})$ which satisfies the *DLR equation*

$$\mu\left(\,\cdot \mid \mathcal{F}_{\Lambda^c}\right)(\omega) = \mu^\omega_{\Lambda,\lambda,h}(\,\cdot\,) \qquad \mu\text{-a.a.}\omega \ \ (\Lambda \Subset \mathbb{Z}^2)$$

is said to be a *Gibbs measure with parameter* (λ, h). The set of all Gibbs measures with parameter (λ, h) is denoted by $\mathcal{G}(\lambda, h)$. It is well-known that $\mathcal{G}(\lambda, h)$ is a non-empty compact convex set. We write $\mathcal{G}_{\text{ex}}(\lambda, h)$ for the set of all extremal Gibbs measures. (For the general properties of Gibbs measures, we refer to [4] or [11].)

Russo [12] introduced the infinite cluster method for studying the phase structure of the 2D Ising model, which is the key step to a final answer ([1], [9]). In [5], the structure of phases is described in terms of percolation and possible extensions