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STOCHASTIC INTEGRALS IN ADDITIVE PROCESSES AND APPLICATION TO SEMI-LÉVY PROCESSES

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1. Introduction

A stochastic process $\{X_t : t \geq 0\}$ on $\mathbb{R}^d$ is called an additive process if it has independent increments and if it is continuous in probability with cadlag paths and $X_0 = 0$. It is called a Lévy process if, in addition, it has stationary increments. Path behaviors and distributional properties of Lévy processes are deeply analyzed (see [1], [19]). Concerning additive processes, the Lévy–Itô decomposition of paths is known in complete generality. But, in order to get further results, we have to restrict our study to some special classes. Examples are the class of selfsimilar additive processes introduced in [18] and the class of semi-selfsimilar additive processes in [12]. Another interesting class is that of semi-Lévy processes, that is, additive processes with semi-stationary (sometimes called periodically stationary) increments. In order to analyze distributional properties of processes of these classes, it is important to treat stochastic integrals (of nonrandom integrands) based on additive processes. Keeping in mind this application, we study in this paper stochastic integrals based on additive processes and their distributions.

Our study in this paper does not depend on the cadlag property. We define additive processes in law, Lévy processes in law, and semi-Lévy processes in law, dropping the cadlag requirement in their definitions but retaining the requirement of continuity in probability. We will call an additive process in law $\{X_t : t \geq 0\}$ *natural* if the location parameter γ_t in the generating triplet (A_t, ν_t, γ_t) of the distribution of X_t is locally of bounded variation in t. An additive process is natural if and only if it is, at the same time, a semimartingale. This fact is essentially given in Jacod and Shiryaev [5]. Thus we can consider stochastic integrals for natural additive processes as a special case of semimartingale integrals of Kunita and Watanabe [9]. But we will not rely on the theory of semimartingales, but directly define stochastic integrals (of nonrandom functions) and seek the representation of the characteristic functions of their distributions. This is in the same line as the study of independently scattered random measures by Urbanik and Woyczynski [23] and Rajput and Rosinski [15]. We show that a natural additive process in law on $\mathbb{R}^d$ induces an $\mathbb{R}^d$-valued independently scattered random measure, and vice versa. Thus our random measures are $\mathbb{R}^d$-valued, not $\mathbb{R}$-valued as in [23] and [15]. Further, we are interested in construction of random