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CORRECTED ENERGY OF DISTRIBUTIONS ON RIEMANNIAN MANIFOLDS

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1. Introduction

In the mathematical literature there are several functionals which let us measure how the vector fields defined over any Riemannian manifold M^n are *ordered*. We can ask ourselves which are the *optimal* vector fields. In fact, we try to measure how far from being parallel our vector field is. We can also extend this question to distributions.

Gluck, Ziller [5] and Johnson [6], among others, studied the *volume* of unit vector fields. They define the volume of a unit vector field X to be the volume of the submanifold in the unit tangent bundle defined by $X(M)$. For this, we regard the vector field as a map $X: M \to T^1M$ and in T^1M we consider the Sasaki metric. We know [5] that in the ambient manifold S^3 the Hopf vector fields, and no others, minimize this functional. For higher dimensional spheres, we know [6] that the Hopf vector fields are unstable critical points; that is, they are not even local minima.

Wiegmink [8] defined the *total bending* of a unit vector field X. This functional is related to the *energy* of the map $X: M \to T^1M$, as we shall see in Section 3. Brito [1] proved that the Hopf vector fields in S^3 are the only minima of the total bending. Furthermore, he proved a more general result giving an absolute minimum in any dimension of the total bending corrected by the second fundamental form of the orthogonal distribution to the field X. The coefficient of this correction vanishes in dimension 3 and then the corrected total bending agrees with the total bending.

Similarly to the situation for vector fields, the energy of a q-distribution $\mathcal{V}$ in a compact oriented Riemannian manifold M is the energy of the section of the Grassmann manifold of q-planes in M induced by $\mathcal{V}$.

In this paper, we add to the energy the norms of the mean curvatures of $\mathcal{V}$ and its orthogonal distribution (with different weights) introducing in this way the *corrected energy*.

In Theorem 1, we find a lower bound for the corrected energy of a foliation, and

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