

CONJECTURES ON CHARACTER DEGREES FOR THE SIMPLE THOMPSON GROUP

Dedicated to Professor Yukio Tsushima on his sixtieth birthday

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1. Introduction

Let Th be the simple Thompson group. It has order $2^{15} \cdot 3^{10} \cdot 5^3 \cdot 7^2 \cdot 13 \cdot 19 \cdot 31$, and its Schur multiplier and the outer automorphism group are both trivial.

In this paper, we prove that Dade's conjecture for Th is true. Here we mean by saying the conjecture the ordinary form of the conjecture. (see [2].) According to [4], the ordinary form is equivalent to the inductive form if the Schur multiplier and the outer automorphism group are trivial. Moreover, if a defect group of a block is cyclic, then the ordinary form is proved to be true for this block in [2]. Thus, it suffices to treat the primes 2, 3, 5 and 7. The character tables of several subgroups of Th are available. We use those found in ATLAS [1] and GAP library. Also, maximal local subgroups of Th are determined in [12] and [18]. These information are quite useful. In the proof of the conjecture, we use a reduction theorem, which is a very special case of the one proved in §16 of [3].

Let G be a finite group and p a prime. Quite recently, a new type of a conjecture, which concerns the p' -part of character degrees, is proposed by Isaacs and Navarro [10]. Though the original version considers only height zero characters of G and the normalizer of a defect group, here we prove a version of Dade's type. Namely, we prove that the alternating sum of the numbers of relevant characters of the normalizers of p -chains with any fixed defect is zero. Since we count the number of characters satisfying a certain congruent relation such as $\chi(1)_{p'} \equiv \pm \kappa \pmod{p}$, where $\chi(1)_{p'}$ is the p' -part of $\chi(1)$, it suffices to consider the case of $p \geq 5$. However, for $p = 13, 19$ or 31 , a Sylow p -subgroup of Th is cyclic of prime order. Thus the normalizers of chains are just Th and the normalizer of a Sylow p -subgroup, and moreover, it is known that those p -blocks of the normalizers have only characters of height zero. Hence the assertion of our version of the conjecture is equivalent to that of Isaacs and Navarro, and the latter is proved for cyclic defect group cases in Theorem 2.1 of [10]. Thus when proving the alternating sum version of the conjecture, we treat only the cases of $p = 5$ and 7 in this paper. Our version of the conjecture is proposed in §3 below.