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INTERFACE REGULARITY FOR MAXELL AND STOKES SYSTEMS

TAKAYUKI KOBAYASHI, TAKASHI SUZUKI and KAZUO WATANABE

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1. Introduction

The purpose of the present paper is to study the interface regularity of three dimensional Maxwell and Stokes systems. To our knowledge, not so much regards have been taken in this topic, but actually the solenoidal condition provides the regularity across interface to a specified component of the unknown vector field.

Let $\Omega \subset \mathbf{R}^3$ be a bounded domain with Lipschitz boundary $\partial\Omega$, and $\mathcal{M} \subset \mathbf{R}^3$ be a C^2 hypersurface cutting Ω transversally. Then, it holds that

$$\begin{aligned} &\mathcal{M} \cap \Omega \neq \emptyset \\ &\Omega = \Omega_+ \cup (\Omega \cap \mathcal{M}) \cup \Omega_- \quad \text{(disjoint union)} \end{aligned} \tag{1}$$

with the open subsets $\Omega_\pm$ of Ω. First, we take the Maxwell system in magnetostatics,

$$\left.\begin{aligned} \nabla \times B = J \\ \nabla \cdot B = 0 \end{aligned}\right\} \quad \text{in} \quad \Omega_\pm, \tag{2}$$

where $B = (B^1(x), B^2(x), B^3(x))$ and $J = (J^1(x), J^2(x), J^3(x))$ stand for the three dimensional vector fields, indicating the magnetic field and the total current density, respectively. Here and henceforth, $\nabla = {}^T(\partial_1, \partial_2, \partial_3)$ denotes the gradient operator and $\times$ and $\cdot$ are the outer and the inner products in $\mathbf{R}^3$, so that $\nabla\times$ and $\nabla\cdot$ are the operations of the rotation and the divergence, respectively.

In the context of magnetoencephalography, Suzuki, Watanabe, and Shimogawara [2] studied the case when the interface is given by the boundary ∂D of a smooth bounded domain $D \subset \mathbf{R}^3$. Namely, from the properties of the layer potential, it showed that if J is piecewise continuous on $\mathbf{R}^3 \setminus \partial D$ and system (2) has a solution $B \in C(\mathbf{R}^3)^3 \cap C^1(\mathbf{R}^3 \setminus \partial D)^3$ for $\Omega_- = D$ and $\Omega_+ = \mathbf{R}^3 \setminus D$, then

$$[\nabla(n \cdot B)]_-^+ = 0 \quad \text{on} \quad \partial D$$

follows, regardless with the continuity of J across ∂D. Here, n denotes the outer unit