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CAUCHY PROBLEM IN GEVREY CLASSES FOR SOME EVOLUTION EQUATIONS OF SCHRÖDINGER TYPE

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1. Introduction

In this paper the Cauchy problem in Gevrey classes is studied for some partial differential — or, more generally, pseudo-differential — equations of Schrödinger type, that is, for differential equations whose type of evolution is 2 and whose characteristic roots are real. Our aim is to determine some Gevrey index σ for which the well-posedness of the Cauchy problem holds in Gevrey classes of order σ. Such an index depends on the multiplicity of the characteristic roots and on the lower order terms. Our result was obtained in [2] in the special case of differential equations with constant leading coefficients.

2. Notation

Let us first introduce some notation about Gevrey spaces.

If $\sigma \geq 1$, then $\gamma^\sigma(\mathbb{R}^n)$ will denote the class of all the smooth functions f such that:

$$\sup_{\substack{x\in\mathbb{R}^n\\ \alpha\in\mathbb{N}^n}} |\partial_x^\alpha f(x)|.A^{-|\alpha|}\alpha!^{-\sigma} < +\infty$$

for some $A > 0$.

Now we define some Gevrey-Sobolev spaces (compare [4] and [5]). For $\varepsilon > 0$, $\sigma \geq 1$, $k > 0$, let $\mathcal{D}_{L^2}^{\sigma,\varepsilon,k}(\mathbb{R}^n)$ denote the space of all functions f such that $\|e^{\varepsilon\langle D_x\rangle^{1/\sigma}} f\|_k < +\infty$, where $\|.\|_k$ is the usual Sobolev norm in $H^k(\mathbb{R}^n)$. Note that, if $k' < k$ and $\varepsilon' > \varepsilon$, then $\mathcal{D}_{L^2}^{\sigma,\varepsilon,k}(\mathbb{R}^n) \subset \mathcal{D}_{L^2}^{\sigma,\varepsilon',k'}(\mathbb{R}^n)$. In this paper the space of the functions belonging to $\mathcal{D}_{L^2}^{\sigma,\varepsilon,0}(\mathbb{R}^n)$ for some ε, will be denoted by $\mathcal{D}_{L^2}^{\sigma}(\mathbb{R}^n)$. Let $\varepsilon(t)$ be a positive function of t, $t \in [-T, T]$. If $u(t, .) \in \mathcal{D}_{L^2}^{\sigma,\varepsilon(t),k}(\mathbb{R}^n)$, for every $t \in [-T, T]$, let us denote $\|e^{\varepsilon(t)\langle D_x\rangle^{1/\sigma}} u(t, x)\|_k$ by $|||u(t)|||_{\varepsilon(t),\sigma,k}$.

Let us now give some notation about pseudo-differential operators. We shall denote by S_σ^p the class of the pseudo-differential operators $s(x, D_x)$ whose symbol $s(x, \xi)$