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LAMINATION OF THE MODULI SPACE OF CIRCLES AND THEIR LENGTH SPECTRUM FOR A NON-FLAT COMPLEX SPACE FORM

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1. Introduction

A smooth curve $\gamma\colon \mathbb{R} \to M$ parametrized by its arclength on a complete Riemannian manifold M is called a *circle* of *geodesic curvature* κ if it satisfies the differential equation $\nabla_{\dot\gamma}\nabla_{\dot\gamma}\dot\gamma(t) = -\kappa^2\dot\gamma(t)$. Here κ is a non-negative constant and $\nabla_{\dot\gamma}$ denotes the covariant differentiation along γ with respect to the Riemannian connection on M. When $\kappa = 0$, as γ is parametrized by its arclength, this equation is equivalent to the equation of geodesics. In this paper we study the set of congruence classes of circles on a non-flat complex space form, which is either a complex projective space $\mathbb{C}P^n$ or a complex hyperbolic space $\mathbb{C}H^n$. We call two circles γ_1 and γ_2 on M are *congruent* if there exist an isometry φ of M and a constant t_0 satisfying $\gamma_1(t) = \varphi \circ \gamma_2(t + t_0)$ for all t. We denote by $\mathrm{Cir}(M)$ the set of all congruence classes of circles on M.

In the preceding papers [5] and [3], we studied length spectum of circles on non-flat complex space forms. We call a circle γ *closed* if it satisfies $\gamma(t) = \gamma(t + t_c)$ for every t with some positive constant t_c. The minimum positive t_c with this property is called the *length* of γ and is denoted by $\mathrm{length}(\gamma)$. For an open circle γ, a circle which is not closed, we set $\mathrm{length}(\gamma) = \infty$. The *length spectrum* $\mathcal{L}\colon \mathrm{Cir}(M) \to \mathbb{R} \cup \{\infty\}$ of circles is defined by $\mathcal{L}([\gamma]) = \mathrm{length}(\gamma)$, where $[\gamma]$ denotes the congruence class containing γ. In these papers [5], [3], we find that the moduli spaces $\mathrm{Cir}(\mathbb{C}P^n)$ and $\mathrm{Cir}(\mathbb{C}H^n)$ of circles on non-flat complex space forms have a natural lamination structure: If we restrict the length spectrum $\mathcal{L}$ on each leaf, it is continuous. In the first half of this paper we study the phenomenon of circles at the boundary of each leaf. For a sequence $\{\sigma_\iota\}$ of closed curves $\sigma_\iota\colon S^1 = [0,1]/\sim\ \to M$ on M we shall call $\lim \sigma_\iota$ its limit curve if it exists. We study this lamination from the viewpoint of limit curves of circles.

The second half of this paper is devoted to add some resluts on length functions of circles on non-flat complex space forms. As two circles have the same geodesic

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