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THE FUNDAMENTAL GROUP OF THE COMPLEMENT OF THE BRANCH CURVE OF $T \times T$ IN $\mathbb{C}^2$

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1. Background

The concentration on the fundamental group of a complement of a branch curve of an algebraic surface X with respect to a generic projection onto $\mathbb{CP}^2$, leads us to the computation of $\pi_1(X_{Gal})$ the fundamental group of the Galois cover of X with respect to this generic projection. Galois covers are surfaces of a general type.

Bogomolov conjectured that the Galois covers corresponding to generic projections of algebraic surfaces to $\mathbb{CP}^2$ have infinite fundamental groups.

In [7] we justify Bogomolov's conjecture by proving that $\pi_1(T \times T)_{Gal}$ is an infinite group.

In order to compute $\pi_1(T \times T)_{Gal}$, we have to enclode the braid monodromy factorization of the branch curve S of $T \times T$. Then we have to apply the van Kampen Theorem on the factors in the factorization in order to get relations for $\pi_1(\mathbb{C}^2 - S, *)$ the fundamental group of the complement of S in $\mathbb{C}^2$.

The fundamental group of the Galois cover X_{Gal} is known to be a quotient of a certain subgroup of the fundamental group of the complement of S.

We recall shortly the computations from [6].

Let $X = T \times T$ be an algebraic surface (where T is a complex torus) embedded in $\mathbb{CP}^5$, and $f\colon X \to \mathbb{CP}^2$ be a generic projection. We degenerate X to a union of 18 planes X_0([6, Section 3]). We numerate the lines and vertices as shown in Fig. 1.

We have a generic projection $f_0\colon X_0 \to \mathbb{CP}^2$. We get a degenerated branch curve S_0 which is a line arrangement and compounds nine 6-points. We regenerate each 6-point separately.

We concentrate for example in a regeneration in a neighbourhood of V_2. We consider the local numeration of lines meeting at V_2 ([6, Figure 6]). First, the diagonal lines 4 and 5 become conics which are tangent to the lines 2, 3 and 1, 6 respectively, see Fig. 2.

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