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ON THE STABLE CLASSIFICATION OF SPIN FOUR-MANIFOLDS

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1. Introduction

The stable classification of closed connected topological respectively smooth four-manifolds (with orientation or spin structure) via bordism theory is a very nice result in topology of manifolds, and can be found in [11] and [23]. Here *stably* means that one allows additional connected sums with copies of $\mathbb{S}^2 \times \mathbb{S}^2$ on both sides. In [19] the closed oriented 4-manifolds with finitely presentable fundamental group π were classified modulo connected sum with simply connected closed 4-manifolds. More precisely, the stable equivalence classes of these manifolds are bijective to the quotient $H_4(B\pi;\mathbb{Z})/(\operatorname{Aut}\pi)_*$ via the map $M \to f_*[M]$, where $[M] \in H_4(M)$ is the fundamental class, and $f\colon M \to B\pi$ is the classifying map for the universal covering of M (see [19, Theorem 1]). The proof of this theorem is based on some facts concerning the cobordism groups $\Omega_4(M)$, $\Omega_4(B\pi)$, and Ω_4 (see for example [7] and [28]). Recently, this result has been extended to the non-orientable case in [18] at least for abelian fundamental groups.

The aim of the present paper is to study the stable classification of closed connected oriented spin smooth 4-manifolds by using techniques of Kervaire-Milnor surgery, as explained for example in [4], [5], [6], and [20]. Then we reproduce a nice result of Kurazono and Matumoto [19] for such manifolds under the assumption that the fundamental group is finitely presentable and has vanishing second and third homology with $\mathbb{Z}_2$-coefficients.

Let $\mathcal{M}_\pi$ (resp. $\mathcal{M}_\pi^{\mathrm{Spin}}$) be the set of closed connected oriented smooth (resp. spin) 4-manifolds with finitely presentable fundamental group π, which are considered up to (resp. spin) stable equivalence. We say that two manifolds in $\mathcal{M}_\pi$ (resp. $\mathcal{M}_\pi^{\mathrm{Spin}}$) are (resp. *spin*) *stably equivalent* if they become diffeomorphic (resp. spin preserving diffeomorphic) after taking connected sums with copies of $\mathbb{S}^2 \times \mathbb{S}^2$ and $\mathbb{S}^2 \underset{\sim}{\times} \mathbb{S}^2$ (resp. $\mathbb{S}^2 \times \mathbb{S}^2$) on both sides. The first result of the paper is the following

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