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CONNECTIVE COVERINGS OF SPACES OF HOLOMORPHIC MAPS

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1. Introduction

For each integer $d \geq 0$, we denote by $\mathrm{Hol}_d(S^2, \mathbb{C}\mathrm{P}^n)$ the space consisting of all holomorphic maps $S^2 \to \mathbb{C}\mathrm{P}^n$ of degree d. The corresponding space of continuous maps is denoted by $\mathrm{Map}_d(S^2, \mathbb{C}\mathrm{P}^n)$. We also denote by $\mathrm{Hol}_d^*(S^2, \mathbb{C}\mathrm{P}^n)$ (resp. $\Omega_d^2\mathbb{C}\mathrm{P}^n$) the subspace of $\mathrm{Hol}_d(S^2, \mathbb{C}\mathrm{P}^n)$ (resp. $\mathrm{Map}_d(S^2, \mathbb{C}\mathrm{P}^n)$) consisting of all maps $f \in \mathrm{Hol}_d(S^2, \mathbb{C}\mathrm{P}^n)$ which preserve the base-points. The space of holomorphic maps are of interest both from a classical and modern point of view (e.g. [1], [3], [6]). It is an elementary and fundamental fact that $\mathrm{Hol}_d(S^2, \mathbb{C}\mathrm{P}^n)$ and $\mathrm{Hol}_d^*(S^2, \mathbb{C}\mathrm{P}^n)$ are connected spaces. If $n = 1$, the fundamental groups of these spaces are $\mathbb{Z}/2d$ and $\mathbb{Z}$, repectively ([7], [12]); if $n \geq 2$, these spaces are simply connected and $2(n-1)$-connected, respectively. The following more general result was obtained by G. Segal:

Theorem 1.1 ([12]). *If*

$$\begin{cases} i_d\colon \mathrm{Hol}_d(S^2, \mathbb{C}\mathrm{P}^n) \to \mathrm{Map}_d(S^2, \mathbb{C}\mathrm{P}^n) \\ \tilde{i}_d\colon \mathrm{Hol}_d^*(S^2, \mathbb{C}\mathrm{P}^n) \to \Omega_d^2\mathbb{C}\mathrm{P}^n \end{cases}$$

are inclusion maps, i_d and $\tilde{i}_d$ are homotopy equivalences up to dimension $D(d,n) = (2n-1)d$.

REMARK. The map $f\colon X \to Y$ is said to be *a homotopy equivalence up to dimension N* if $f_*\colon \pi_k(X) \to \pi_k(Y)$ is bijective when $k < N$ and surjective when $k = N$.

The principal motivation of this paper derives from the work of Segal ([12]), in which he describes the homotopy types of $\mathrm{Hol}_d(S^2, \mathbb{C}\mathrm{P}^n)$ and $\mathrm{Hol}_d^*(S^2, \mathbb{C}\mathrm{P}^n)$ from the point of view of the infinite dimensional Morse theoretical principle by using a technique of scanning maps ([8], [9], [12]). Now the homotopy types of $\mathrm{Hol}_d^*(S^2, \mathbb{C}\mathrm{P}^n)$ were studied well by several authors ([3], [9], [10]). So in this paper we shall study the homotopy types of $\mathrm{Hol}_d(S^2, \mathbb{C}\mathrm{P}^n)$. We identify $S^2 = \mathbb{C} \cup \infty$ and consider the evaluation fibration sequence $\mathrm{Hol}_d^*(S^2, \mathbb{C}\mathrm{P}^n) \xrightarrow{j_d} \mathrm{Hol}_d(S^2, \mathbb{C}\mathrm{P}^n) \xrightarrow{ev} \mathbb{C}\mathrm{P}^n$, where the map ev is given by $ev(f) = f(\infty)$ for $f \in \mathrm{Hol}_d(S^2, \mathbb{C}\mathrm{P}^n)$.