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TRANSFORMS OF CURRENTS BY MODIFICATIONS AND 1-CONVEX MANIFOLDS

LUCIA ALESSANDRINI and GIOVANNI BASSANELLI

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1. Introduction

Let X' and X be complex manifolds (not compact, a priori), and $X' \stackrel{\alpha}{\to} X$ a proper modification with center Z and exceptional divisor E, whose irreducible components are $\{E_k\}$. Let Y be an analytic subset of X without irreducible components in Z: then its strict (proper) transform Y' is a well-defined analytic subset of X'. In particular, when D is a complex hypersurface of X, we can define the strict transform D' and also the total transform

$$\alpha^* D = D' + \sum_k n_k E_k, \quad n_k \geq 0. \tag{1.1}$$

In the first part of this paper we shall extend these notions to the case of currents on X, and ask for the existence and uniqueness of strict and total transforms.

We can look for a strict transform T' of a current T on X (of every bidegree) when T is of order zero and $\chi_Z T = 0$ (see Definition 3.1); moreover, if a strict transform exists, it is unique (see Proposition 3.2).

On the other hand, to define the total transform $\alpha^* T$ of a current T on X (Definition 3.3), T must be "closed" in some sense: in fact, the idea is that if φ is a smooth form on X, cohomologous to T, then $\alpha^* T$ should be cohomologous to $\alpha^* \varphi$. The classical case is that of d-closed currents, while the most general context seems to be that of $\partial\overline{\partial}$-closed currents (i.e. pluriharmonic currents); moreover, we would like to generalize (1.1) as:

$$\alpha^* T = T' + L \tag{1.2}$$

where L is a current supported on E. As for existence results, since we have to estimate locally the mass of $T_\alpha := (\alpha|_{X'-E})^{-1}_*(T|_{X-Z})$, we shall assume $T \geq 0$ (in the sense of Lelong).

But notice that defining a "good" total transform, besides bidegree $(1,1)$, seems hopeless: for instance, if Y is a line through the origin in $\mathbf{C}^3$ and $X' \stackrel{\alpha}{\to} X := \mathbf{C}^3$ is the blow-up with center in the origin, what could be the "true" meaning of $\alpha^* Y$?

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