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ON THE NIELSEN-THURSTON-BERS TYPE OF SOME SELF-MAPS OF RIEMANN SURFACES WITH TWO SPECIFIED POINTS

Dedicated to Professor Hiroki Sato on his 60th birthday

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1. Introduction

1.1. Let S be a hyperbolic Riemann surface of analytically finite type, that is, a hyperbolic Riemann surface obtained by removing n_0 distinct points from a closed Riemann surface of genus g_0 with $2g_0 - 2 + n_0 > 0$. Take n distinct points $p_1, p_2, \dots, p_n$ of S, and set $\dot{S} = S \setminus \{p_1, p_2, \dots, p_n\}$. We consider the group of orientation preserving homeomorphisms ω of S onto itself which satisfy two conditions
(1) $\omega(p_j) = p_j$ for every $j = 1, 2, \dots, n$, and
(2) ω is isotopic to the identity self-map id of S.
We factor this group by the normal subgroup of homeomorphisms of S onto itself that are isotopic to the identity as self-maps of $\dot{S}$. Denote the factor group by

$$\mathrm{Isot}(S, \{p_1, p_2, \dots, p_n\}), \text{ or } \mathrm{Isot}(S, n).$$

1.2. The purpose of this paper is to classify the elements of $\mathrm{Isot}(S, n)$ in the case of $n = 2$. For $n = 1$, it is studied by Kra [10]. Our problem and form of the solution are suggested by his beautiful theorem (Theorem 2 of Kra [10]).

Every element $[\omega] \in \mathrm{Isot}(S, n)$ induces canonically an element $\langle \omega|_{\dot{S}} \rangle$ of the Teichmüller modular group $\mathrm{Mod}(\dot{S})$. Namely, for the Teichmüller space $T(\dot{S})$ of $\dot{S}$, $\langle \omega|_{\dot{S}} \rangle$ is an biholomorphic automorphism of $T(\dot{S})$ given by $\langle \omega|_{\dot{S}} \rangle([S, f, R]) = [S, f \circ \omega^{-1}, R]$ for all $[S, f, R] \in T(\dot{S})$. Since the correspondence $\mathrm{Isot}(S, n) \ni [\omega] \mapsto \langle \omega|_{\dot{S}} \rangle \in \mathrm{Mod}(\dot{S})$ is injective, we can classify $[\omega]$ by a classification for the elements of $\mathrm{Mod}(\dot{S})$.

For our classification, we use the following one due to Bers [1] for the elements of $\mathrm{Mod}(\dot{S})$. Let $d_{T(\dot{S})}$ be the Teichmüller distance on $T(\dot{S})$, and set

$$a(\chi) = \inf_{\tau \in T(\dot{S})} d_{T(\dot{S})}(\tau, \chi(\tau)).$$

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