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SCHOTTKY GROUPS AND BERS BOUNDARY OF TEICHMÜLLER SPACE

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1. Introduction

We will show that every Kleinian group on the Bers boundary of the Teichmüller space is an algebraic limit of a sequence of Schottky groups. This claim has already proved by Otal [26], but our argument is completely different from that of Otal. In this paper, we extend the action of the mapping class group on a Bers slice to that on a wider class of Kleinian groups. We obtain a sufficient condition for the action of the mapping class group to be continuous at a given point and show that the orbit of every maximal cusp is dense in the Bers boundary.

Here, we explain the fundamental idea of extending the action of the mapping class group. Let S be an oriented compact surface possibly with boundary ∂S, and let $T(S)$ be the Teichmüller space of complete hyperbolic structures on the interior of S with finite area. Let $R(S)$ be the space of conjugacy classes $[\rho, G]$ of representations $\rho\colon \pi_1(S) \to G \subset \mathrm{PSL}_2(\mathbf{C})$ of $\pi_1(S)$ which map each component of the boundary ∂S to parabolic elements. The subspace $QF(S)$ of $R(S)$ consisting of discrete faithful representations whose images are quasi-Fuchsian groups is naturally identified with a product of Teichmüller spaces $T(S) \times T(\overline{S})$, where $\overline{S}$ denotes S with its orientation reversed. We denote the canonical homeomorphism by

$$Q\colon T(S) \times T(\overline{S}) \to QF(S).$$

The mapping class group $\mathrm{Mod}(S)$ of S naturally acts on $T(S)$ and $T(\overline{S})$ and hence on the Bers slice $B_X = Q(\{X\} \times T(\overline{S}))$ by

$$Q(X, \overline{Y}) \mapsto Q(X, \sigma\overline{Y})$$

for $(X, \overline{Y}) \in T(S) \times T(\overline{S})$ and $\sigma \in \mathrm{Mod}(S)$. A crucial observation is that the representation $Q(X, \sigma\overline{Y})$ has another description as follows;

$$Q(X, \sigma\overline{Y}) = Q(\sigma^{-1}X, \overline{Y}) \circ {\sigma_*}^{-1},$$

where σ_* is the group isomorphism of $\pi_1(S)$ induced by σ. The right-hand side of the above equation suggests us a possibility to define the action of $\mathrm{Mod}(S)$ even when $\overline{Y}$