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CLOSABILITY OF POSITIVE SYMMETRIC BILINEAR FORMS WITH APPLICATIONS TO CLASSICAL AND STABLE FORMS ON FINITE AND INFINITE DIMENSIONAL STATE SPACES

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1. General framework and basic definitions

In the literature concerning Dirichlet forms and its applications, closability plays a crucial role. In fact, closedness is one of the defining properties of a Dirichlet form. According to this, a number of closability criterions are known in particular cases. An important question is under which conditions closability is kept after changing the reference measure.

M. Fukushima, K. Sato, and S. Taniguchi [5] treated this problem for a regular Dirichlet form $(\mathcal{E}, D(\mathcal{E}))$ which is defined on a locally compact separable metric state space. Under technical conditions on some core $\mathcal{C} \subseteq D(\mathcal{E})$, they presented a complete solution if the Dirichlet form is either irreducible or transient. An earlier paper dealing with this subject is M. Röckner and N. Wielens [13]. Related results on Lusinean separable metric spaces were published in I. Shigekawa and S. Taniguchi [16].

The aim of this paper is to give general analytical conditions in order to keep closability when turning to a new reference measure. One particular purpose is to present an extension of an assertions in [5] (namely, Corollary 4.2) within a purely measure theoretic framework, i.e., the state space $(E, \mathcal{B})$ is just a measurable space. In particular, the set $\mathcal{C}$ is defined exclusively in terms of the initial form $(\mathcal{E}, D(\mathcal{E}))$ on $L^2(E, \mu)$. The main results are Theorems 2.3, 2.4, and 2.5.

We proceed to give some basic definitions.

Definition 1.1. Let $(H, \|.\|_H)$ be a separable Hilbert space and let $\mathcal{F}$ be a dense subset of H.
(i) A positive symmetric bilinear form (p.s.b.f.) $\mathcal{E}$ defined on $\mathcal{F}$ is said to be *closed* if $\mathcal{F}$, equipped with the $(\mathcal{E}_1)^{1/2}$-norm $\|f\|_{\mathcal{E}_1} := (\|f\|_H^2 + \mathcal{E}(f, f))^{1/2}$, is a Hilbert space.
(ii) Let $\mathcal{C}$ be a subspace of H. We say that a p.s.b.f. $(\mathcal{E}, \mathcal{C})$ is *pre-closable* on H if, for all sequences $u_n \in \mathcal{C}$, $n \in \mathbb{N}$, which are $\mathcal{E}$-Cauchy (i.e., $\mathcal{E}(u_n - u_m, u_n -$

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