El Goul, J.
Osaka J. Math.
40 (2003), 469–491

LOGARITHMIC JETS AND HYPERBOLICITY

Jawher El GOUL

(Received September 26, 2001)

0. Introduction

In 1970 S. Kobayashi [21] posed the following problem: Is it true that the complement of a generic hypersurface of degree $d \geq e(n)$ in $\mathbb{P}^n_{\mathbb{C}}$ is hyperbolic for some number $e(n)$? Is this true for $e(n) = 2n + 1$? Later Green [15] for $n = 2$ and Zaidenberg [35] for arbitrary n proved that for $d \leq 2n$ such complements contain $\mathbb{C}^\star$ and so are not hyperbolic.

In this paper we study the case of complements of smooth curves in $\mathbb{P}^2_{\mathbb{C}}$ where, for $d \geq 4$ this problem is equivalent to the nonexistence of nonconstant entire curves by Brody Reparametrization Lemma [4]. When the curve has many components, this problem had been studied by many authors, see [12, 13] for a complete bibliography and the study of the case of three components (see also the recent work [2]). In the smooth case, the first example of hyperbolic complement was constructed by Azukawa and Suzuki in [1] (for even degree $d \geq 30$), then Zaidenberg in [36] showed that examples exist for all $d \geq 5$.

The first positive answer to this problem (for $n = 2$) was given in the work of Siu and Yeung [32] though the bound they obtain is quite high. Their method is rather involved and consists of an explicit construction of special second order differential operators on an associated surface in $\mathbb{P}^3_{\mathbb{C}}$ ramified over $\mathbb{P}^2_{\mathbb{C}}$. This was done by an imitation of the construction of holomorphic 1-forms on Riemann surfaces and a clever reduction of the problem to a resolution of linear systems. Those operators are such that their pullbacks by the lifting of every entire curve must vanishes identically. This follows from an Ahlfors-Schwarz type result.

In [10], after studying the compact analogue of the above conjecture and proving that a generic suface of degree $d \geq 21$ in $\mathbb{P}^3_{\mathbb{C}}$ is hyperbolic, Demailly and the author, using the same covering trick, obtained the bound 21 also for complementary of curves in $\mathbb{P}^2_{\mathbb{C}}$ (see also [25]). This is made by using the whole force of Demailly's jet bundles introduced in [8] and a result of McQuillan on holomorphic foliations [24].

In this paper, we obtain the bound 15 for the complement of a very generic curve of degree d in $\mathbb{P}^2_{\mathbb{C}}$. Here, the terminology "very generic" refers to complements of countable unions of proper algebraic subsets of the parameter space. Our main theorem is the following