Nakagawa, M.
Osaka J. Math.
40 (2003), 429–448

THE SPACE OF LOOPS ON THE EXCEPTIONAL LIE GROUP E_6

MASAKI NAKAGAWA

(Received November 15, 2001)

1. Introduction

Let G be a compact 1-connected simple Lie group and ΩG the space of loops on G. As is well known ΩG is a homotopy commutative H-space and its integral homology $H_*(\Omega G)$ has no torsion and no odd dimensional part ([2]). Therefore $H_*(\Omega G)$ becomes a commutative Hopf algebra over the integers $\mathbb{Z}$. In [3] R. Bott introduced a "generating variety" and determined the Hopf algebra structure of $H_*(\Omega G)$ explicitly for $G = SU(n)$, $\mathrm{Spin}(n)$ and G_2. In [11] T. Watanabe determined $H_*(\Omega F_4)$ in a similar way. On the other hand A. Kono and K. Kozima determined $H_*(\Omega Sp(n))$ by different method using the Bott periodicity ([6]).

In this paper we carry out the Bott's program for $G = E_6$, where E_6 is the compact 1-connected exceptional Lie group of rank 6 and determine the Hopf algebra structure of $H_*(\Omega E_6)$ explicitly.

Let ψ be the coproduct of $H_*(\Omega G)$ induced by the diagonal map $\Omega G \longrightarrow \Omega G \times \Omega G$. To avoid the cumbersome notation, following [11] we introduce a map $\tilde{\psi}\colon H_*(\Omega G) \longrightarrow H_*(\Omega G) \otimes H_*(\Omega G)$ satisfying

$$\psi(\sigma) - (\sigma \otimes 1 + 1 \otimes \sigma) = \tilde{\psi}(\sigma) + T\tilde{\psi}(\sigma) \quad \text{for} \quad \sigma \in H_*(\Omega G)$$

where $T\colon H_*(\Omega G) \otimes H_*(\Omega G) \longrightarrow H_*(\Omega G) \otimes H_*(\Omega G)$ is defined by

$$T(\sigma \otimes \tau) = \begin{cases} \tau \otimes \sigma & \text{for} \quad \sigma \neq \tau, \\ 0 & \text{for} \quad \sigma = \tau. \end{cases}$$

Note that $\tilde{\psi}(\sigma) = 0$ if and only if $\sigma \in PH_*(\Omega G)$, where $PH_*(\Omega G)$ denotes the primitive module of the Hopf algebra $H_*(\Omega G)$.

Then our main results are stated as follows:

Theorem 1.1. *The Hopf algebra structure of $H_*(\Omega E_6)$ is given as follows*:
(i) *As an algebra*

$$H_*(\Omega E_6) = \mathbb{Z}[\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_7, \sigma_8, \sigma_{11}]/({\sigma_1}^2 - 2\sigma_2, \sigma_1\sigma_2 - 3\sigma_3)$$

where $\deg(\sigma_i) = 2i$.