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$\mathbb{Z}_n$-EQUIVARIANT GOERITZ MATRICES FOR PERIODIC LINKS

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1. Introduction

A link $l^{(n)}$ in S^3 is said to have *period* $n (n \geq 2)$ if there is an n-periodic homeomorphism ϕ from S^3 onto itself such that $l^{(n)}$ is invariant under ϕ and the fixed point set $\tilde{f}$ of the $\mathbb{Z}_n$-action induced by ϕ is homeomorphic to a 1-sphere in S^3 disjoint from $l^{(n)}$. By the positive solution of the Smith conjecture [10], $\tilde{f}$ is unknotted and so the homeomorphism ϕ is conjugate to one point compactification of the $(2\pi/n)$-rotation about the z-axis in $\mathbb{R}^3$. Hence the quotient map $\pi\colon S^3 \to S^3/\mathbb{Z}_n$ is an n-fold branched cyclic cover branched along $\pi(\tilde{f}) = f$, and $l = \pi(l^{(n)})$ is also a link in the orbit space $S^3/\mathbb{Z}_n \cong S^3$, which is called the *factor link* of $l^{(n)}$.

There are several studies about the relationship between polynomial invariants of $l^{(n)}$ and those of l [5, 11, 14, 15, 16], and also some numerical invariants [3, 4, 9, 13] (see also references therein). In particular, Gordon-Litherland-Murasugi [4] gave a necessary congruence condition mod 4 on the signature of a knot in S^3 for it to have odd prime power period n, by using a $\mathbb{Z}_n$-invariant Hermitian form.

Now let $l = k_1 \cup \cdots \cup k_\mu$ be an oriented link in S^3 of μ components and let f be the oriented trivial knot such that $l \cap f = \emptyset$. For any integer $n \geq 2$, let $\pi\colon S^3 \to S^3$ be the n-fold branched cyclic cover branched along f. We denote the preimage $\pi^{-1}(l)$ and $\pi^{-1}(k_i)$ by $l^{(n)}$ and $k_i^{(n)}$, respectively. Then $k_i^{(n)} = k_{i1} \cup \cdots \cup k_{i\nu_i}$ is a link of ν_i components, where ν_i is the greatest common divisor of n and $\lambda_i = Lk(k_i, f)$, the *linking number* of k_i and f. We give an orientation to $k_i^{(n)}$ inherited from k_i. Then $l^{(n)} = k_1^{(n)} \cup \cdots \cup k_\mu^{(n)} = k_{11} \cup \cdots \cup k_{1\nu_1} \cup \cdots \cup k_{\mu 1} \cup \cdots \cup k_{\mu\nu_\mu}$ is an oriented n-periodic link in S^3 with l as its factor link. Throughout this paper we call such an oriented link $l^{(n)}$ the *n-periodic covering link over $l_1 = l \cup f$*. Notice that every link in S^3 with cyclic period arises in this manner.

Section 2 of the present paper reviews the definitions of Goeritz matrix for a link and its invariants. In Section 3, we characterize a $\mathbb{Z}_n$-equivariant Goeritz matrix for an n-periodic covering link $l^{(n)}$ in terms of its factor link $l \cup f$. In Section 4, we derive a necessary congruence condition mod 4 on the signature of a link for it to be an n-periodic covering link over a certain link. In Section 5, we give a congruence mod p between the reduced Alexander polynomial of an n-periodic covering link $l^{(n)}$ with

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