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FINITENESS OF CONFORMAL BLOCKS OVER COMPACT RIEMANN SURFACES

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Introduction

We study conformal blocks (the space of correlation functions) over compact Riemann surfaces associated to vertex operator algebras which are the sum of highest weight modules for the underlying Virasoro algebra. Under a fairly general condition, for instance, C_2-finiteness, we prove that conformal blocks are finite-dimensional. This, in particular, shows the finiteness of conformal blocks for many well-known conformal field theories including WZNW model and the minimal model.

In [1] we showed that conformal blocks over the *projective line* associated to a vertex operator algebra (VOA) V are finite-dimensional if modules for V satisfy some finiteness condition. In this paper we generalize these results to conformal blocks over any *compact Riemann surfaces*. More precisely we will prove that if V-modules of our concern as well as V are C_2-finite then corresponding conformal blocks are finite-dimensional. The main reason why we need C_2-finiteness of V in this case is caused by Weierstrass gaps, i.e., we are not able to find meromorphic differentials with poles of some exceptional orders.

Though in this paper the notion of conformal blocks are defined in a purely mathematical way, the definition goes back to the notion of correlation functions in conformal field theory (CFT) initiated by [4]. CFT's are supposed to have at least two properties, one of which is the finiteness of conformal blocks, and the other is the factorization property; the latter enables us to determine the dimension of conformal blocks by fusion rules (the space of 3-point correlation functions or its dimension). Like other objects in physics every CFT has its own symmetry group (Lie algebra): affine Lie algebras for WZNW model and the Virasoro algebra for the minimal model, for instance. We will study "general" CFT's, where "general" means that the symmetry is described by a VOA. Such CFT's were first proposed and studied by Zhu [21], however two main issues, i.e., finiteness of conformal blocks and the factorization theorem of these CFT's were left open.

We should point out two main differences between our general CFT's and the

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