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THE EQUIVARIANT WHITEHEAD GROUPS OF SEMIALGEBRAIC G-SETS

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1. Introduction

Throughout this paper, the base field is the real numbers $\mathbb{R}$ and all semialgebraic maps are assumed to be continuous. For general terminology and the theory of semialgebraic sets we refer the reader to [1].

Let G be a compact semialgebraic group. One can see easily that every compact semialgebraic group has a Lie group structure, and conversely, every compact Lie group has a semialgebraic group structure. A *semialgebraic representation* of G is, by definition, a semialgebraic homomorphism $\rho\colon G \to \mathrm{GL}(n, \mathbb{R})$ for some n. In this case $\mathbb{R}^n$ equipped with the linear action of G via ρ is denoted by $\mathbb{R}^n(\rho)$ and called a *semialgebraic representation space* of G. A *semialgebraic G-set* is a G-invariant semialgebraic set in some finite dimensional semialgebraic representation space of G. One may define a semialgebraic G-set as a semialgebraic set with a semialgebraic action of G, but two definitions are equivalent when G is semialgebraically isomorphic to a semialgebraic subgroup of some $\mathrm{GL}(k, \mathbb{R})$, see [17, Thoerem 1.1]. Note that $\mathrm{GL}(k, \mathbb{R})$ is a semialgebraic set in $M_k(\mathbb{R}) \cong \mathbb{R}^{k^2}$ where $M_k(\mathbb{R})$ denotes the set of all $k \times k$ real matrices. A G-equivariant semialgebraic map between semialgebraic G-sets is called a *semialgebraic G-map*.

The simple homotopy theory and the theory of Whitehead torsions have equivariant generalizations in the topological category, see e.g. [6]. In this paper we consider the equivariant generalizations of them to the semialgebraic category. Namely, we define the equivariant Whitehead group of a semialgebraic G-set and the Whitehead torsion of a G-homotopy equivalence between semialgebraic G-sets. Moreover, we prove the semialgebraic invariance of the equivariant Whitehead torsion.

The basic ingredients for the development are the existence of an equivariant semialgebraic G-CW complex structure of a semialgebraic G-set (Proposition 2.2) and equivariant semialgebraic homotopy theory in [16]. We remark that the (equivariant) Whitehead group is defined on a complete (G-) CW complex [6, 13]. However, in general, a semialgebraic G-set has a finite open G-CW complex structure which is not

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