Byun, Y. and Yi, S.
Osaka J. Math.
40 (2003), 81–86

NORMAL EMBEDDING OF SPHERES INTO $\mathbb{C}^n$

YANGHYUN BYUN and SEUNGHUN YI

(Received November 29, 2000)

1. Introduction

The notion of normal submanifold was introduced by J.C. Sikorav ([6]) as a weaker version of Lagrangian submanifold.

Polterovich ([5]) showed that if L is a closed normal non-Lagrangian submanifold of a symplectic manifold M and the Euler characteristic of L vanishes then its displacement energy $e(L)$ vanishes.

The basic notions such as 'normal', 'symplectic', 'weakly Lagrangian' etc. are explained in Section 2 below and the definition of the displacement energy is provided in the later part of this section.

It is well known that S^1 and S^3 are totally real submanifolds of $\mathbb{C}^1$ and $\mathbb{C}^3$, respectively. L. Polterovich ([5]) showed that if L is a totally real submanifold of a symplectic manifold (V,ω) and L is parallelizable then L is normal. So S^1 and S^3 are normal submanifold of $\mathbb{C}^1$ and $\mathbb{C}^3$, respectively. In fact S^1 is a Lagrangian submanifold of $\mathbb{C}^1$ and it follows that it is a normal submanifold. As for S^3 we consider the standard embedding and explicitly construct in Section 4 below the Lagrangian subbundle of $T\mathbb{C}^3|_{S^3}$ which is transverse to the tangent bundle.

The following two theorems are our main results which respectively answer the two questions: (a) Which S^n admits a normal embedding into $\mathbb{C}^n$? (b) When the product of spheres admits a normal embedding into the complex Euclidean space?

Theorem 1.1. *S^n admits a normal embedding into $\mathbb{C}^n$ if and only if n is 1 or 3.*

Theorem 1.2. *$S^{n_1}\times S^{n_2}\times\cdots\times S^{n_k}$, $n_i\geq 1$, $i=1,2,\dots,k$, $k\geq 2$, admits a normal embedding into $\mathbb{C}^{n_1+n_2+\cdots+n_k}$ if and only if some n_i is odd.*

Note that H. Hofer ([3]) defined the *displacement energy* of a subset A of a sym-

1991 *Mathematics Subject Classification* : 53C40.
The author Y. Byun was partially supported by Research Fund of Hanyang University (HY-2000) and the other author S. Yi, by the grant from the Basic Research Program of the Korea Science & Engineering Foundation, R01-2000-000-00004-0.