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ELEMENTARY INTERSECTION NUMBERS ON PUNCTURED SPHERES

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Introduction

According to Thurston, for any analytically finite Riemann surface $\mathcal{R}$, the set $\overline{\mathcal{G}}(\mathcal{R})$ of all projective geodesic laminations in $\mathcal{R}$ can be made into a topological space homeomorphic to a sphere of dimension depending on the topology of $\mathcal{R}$. Understanding the space $\overline{\mathcal{G}}(\mathcal{R})$ is important for various approaches to the Teichmüller space and the mapping class group of $\mathcal{R}$. The space $\overline{\mathcal{G}}(\mathcal{R})$ was then investigated by several authors from many different points of view. See [3–10], [12, 13, 15], and references there in.

In this paper, we consider the space $\overline{\mathcal{G}}_n = \overline{\mathcal{G}}(\Sigma_n)$ for any integer $n \geq 4$, where Σ_n is an n-punctured sphere endowed with a hyperbolic metric. Note that $\overline{\mathcal{G}}_n$ is homeomorphic to a sphere of dimension $2n - 7$.

This work was an attempt to generalize the projective coordinates defined in [3, 4] to an arbitrary $\overline{\mathcal{G}}_n$. This work and that of Keen, Parker and Series [10] are essentially based on cutting sequence technique developed by Birman and Series [2], and complement the works of Masur and Minsky [12, 13].

Let $\mathcal{G}_n$ be the set of all simple closed geodesics on Σ_n. For $n = 4$ or 5, the author has defined a set of projective coordinates for $\mathcal{G}_n$ so that the completion of these coordinates parametrize $\overline{\mathcal{G}}_n$, (see [3, 4]). The coordinates of each $\gamma \in \mathcal{G}_n$ are geometric intersection numbers of γ with $2(n - 3)$ fixed geodesics in $\mathcal{G}_n$, and read off directly from the topology of γ. Moreover, these coordinates have three remarkable applications. First, the geometric intersection number of any two geodesics in $\mathcal{G}_n$ can be formulated explicitly in terms of the corresponding coordinates. Secondly, the coordinates of each $\gamma \in \mathcal{G}_n$ determine a canonical expression of γ as a word in a given set of generators for the fundamental group $\pi_1(\Sigma_n)$. Finally, the coordinates of each $\gamma \in \mathcal{G}_n$ are related to trace polynomials of the transformations corresponding to γ in a family of regular B-groups uniformizing Σ_n.

For an arbitrary $n \geq 5$, following [3, 4], we shall choose $n - 3$ fixed triples $(\gamma_j^1, \gamma_j^2, \gamma_j^3)$ of geodesics in $\mathcal{G}_n$ $(1 \leq j \leq n - 3)$, and compute the geometric intersec-

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