

GENERALIZED TAKAHASHI MANIFOLDS*

MICHELE MULAZZANI and ANDREI VESNIN

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1. Introduction

Takahashi manifolds are closed orientable 3-manifolds introduced in [21] by Dehn surgery with rational coefficients on \mathbf{S}^3 , along the $2n$ -component link \mathcal{L}_n of Fig. 1, which is a closed chain of $2n$ unknotted components. These manifolds have been intensively studied in [10], [11], [17] and [19, 20]. In particular, a topological characterization of all Takahashi manifolds as two-fold coverings of \mathbf{S}^3 , branched over the closure of certain rational 3-string braids, is given in [11] and [19].

A Takahashi manifold is said to be *periodic* when the surgery coefficients have the same cyclic symmetry of order n of the link \mathcal{L}_n , i.e. the coefficients are $p_k/q_k = p/q$ and $r_k/s_k = r/s$ alternately, for $k = 1, \dots, n$. Several important classes of 3-manifolds, such as (fractional) Fibonacci manifolds [7, 11] and Sieradski manifolds [2], represent notable examples of periodic Takahashi manifolds. More generally, all cyclic branched coverings of two-bridge knots of genus one are periodic Takahashi manifolds [10]. A characterization of periodic Takahashi manifolds as n -fold cyclic coverings of the connected sum of two lens spaces, branched over a knot, is given in [17].

In this paper we generalize the family of Takahashi manifolds, as well as periodic Takahashi manifolds, considering surgery along a more general family of links

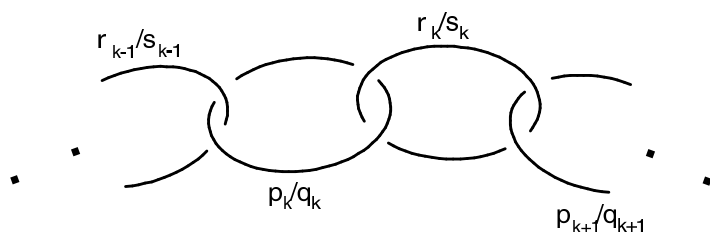


Fig. 1. Surgery presentation for Takahashi manifolds.

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