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THE HYPERBOLIC OPERATORS WITH THE CHARACTERISTICS VANISHING WITH THE DIFFERENT SPEEDS

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0. Introduction

Consider for the partial differential operator

$$P(t, D_t, D_x) := \sum_{j+|\alpha|\le m} a_{j,\alpha}(t) D_t^j D_x^\alpha, \qquad t \in [0, T], \quad x \in \mathbb{R}^n,$$

with coefficients $a_{j,\alpha}(t) \in C^1([0, T])$ the Cauchy problem with the data prescribed at $t = s$,

$$\begin{cases} P(t, D_t, D_x)u(t, x) = f(t, x), & t \in [0, T], \quad x \in \mathbb{R}^n, \\ D_t^j u(s, x) = u_j(x), & j = 0, \dots, m-1, \quad x \in \mathbb{R}^n, \end{cases} \tag{0.1}$$

where $s \in [0, T]$. For the principal symbol $P_m(t, \lambda, \xi)$ of the operator P defined by

$$P_m(t, \lambda, \xi) := \sum_{j+|\alpha|=m} a_{j,\alpha}(t) \lambda^j \xi^\alpha$$

we assume that for all $t \in [0, T]$, $\xi \in \mathbb{R}^n$, the following representation

$$\begin{cases} P_m(t, \lambda, \xi) = \displaystyle\prod_{j=1}^m (\lambda - \lambda_j(t, \xi)), \\ |\lambda_j(t, \xi) - \lambda_k(t, \xi)| \ge C\lambda_j(t)|\xi|, \quad j < k, \end{cases} \tag{0.2}$$

with the real-valued functions $\lambda_j(t, \xi)$, $j = 1, \dots, m$, and with non-negative continuous functions $\lambda_j(t)$, $j = 1, \dots, m$, $\lambda_j \in C([0, T])$, holds. Thus the operator $P(t, D_t, D_x)$ is a hyperbolic operator with the characteristics $\lambda_j(t, \xi)$, $j = 1, \dots, m$. We make also the assumptions

$$\begin{gathered} |\partial_t^k a_{j,\alpha}(t)| \le C\lambda_1(t)\cdots\lambda_{|\alpha|}(t) \left(\frac{\lambda_{|\alpha|}(t)}{\Lambda_{|\alpha|}(t)}\right)^k, \\ 1 \le |\alpha|, \quad j + |\alpha| = m, \quad k = 0, 1, \end{gathered} \tag{0.3}$$