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A CONCENTRATION PHENOMENON AROUND A SHRINKING HOLE FOR SOLUTIONS OF MEAN FIELD EQUATIONS

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1. Introduction

Let Ω be a bounded smooth domain in $\mathbf{R}^2$. In this paper, we consider the following mean field equation in statistical mechanics of point vortices; see [6, 7, 15]:

$$
\text{(P)}\qquad \begin{aligned} -\Delta u &= \rho \frac{e^u}{\int_\Omega e^u} \quad \text{in} \quad \Omega, \qquad \rho > 0 \\ u &= 0 \quad \text{on} \quad \partial\Omega. \end{aligned}
$$

We note that the problem (P) for $\rho < 0$ is treated in [14]; see also [6, 7]. Analogous problems under Neumann boundary conditions are considered in relation to stationary problems of the Keller-Segel system of chemotaxis in [28]. Analogous problems on two-dimensional manifolds are also considered in relation to the prescribed Gauss curvature problem or Chern-Simons-Higgs gauge theory; see [12, 17, 26, 29] and references therein.

It should be also remarked that the following non-linear eigenvalue problem called the Gel'fand problem (see, for example, [3, 32]) also relates to our problem (P):

$$
\text{(G)}\qquad \begin{aligned} -\Delta u &= \lambda e^u \quad \text{in} \quad \Omega, \qquad \lambda > 0 \\ u &= 0 \quad \text{on} \quad \partial\Omega. \end{aligned}
$$

Indeed, every solution of (G) corresponds to the solution of (P) for $\rho = \int_\Omega \lambda \exp u \, dx$.

(P) is the Euler-Lagrange equation of the following functional:

$$
J_\rho(u) = \frac{1}{2}\int_\Omega |\nabla u|^2 - \rho \log \int_\Omega e^u \quad \text{for} \quad u \in H_0^1(\Omega).
$$

Caglioti et al. show the following facts on (P):