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Osaka J. Math.
39 (2002), 181–208

ON CONVERGENCE OF THE FEYNMAN PATH INTEGRAL IN PHASE SPACE

Dedicated to Professor K. Kajitani on his 60th birthday

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(Received June 12, 2000)

1. Introduction

It was an interested and important problem to give the description of quantization, i.e. of passing from classical physical systems to the corresponding quantum ones, from the moment that quantum mechanics came into existence. In the end we succeeded in giving it as follows: Let $\mathcal{L}$ be a Lagrangian function. Then the Hamiltonian function $\mathcal{H}$ is defined through the Legendre transformation of $\mathcal{L}$. The Hamiltonian operator $H(t)$ at time t in quantum mechanics is defined from $\mathcal{H}$. It should be noted that $H(t)$ has ordering ambiguities (cf. [14]). Let f be a probability amplitude at time s. Then its temporal evolution can be given by the solution of the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}u(t) = H(t)u(t), \quad u(s) = f. \tag{1.1}$$

On the other hand Feynman proposed an essentially new description in his famous paper [3] which appeared in 1948. His description is based on the notion of a so-called path integral in configuration space. In 1951 Feynman himself generalized this description, using the notion of a path integral in phase space in [4]. Since then, path integrals in phase space have been discussed by many articles in not only quantum mechanics but also quantum field theory. But it has been pointed out that we have hard difficulties of giving a rigorous meaning to the path integral in phase space. There is even a suggestion that such a path integral can not be defined rigorously. For example see chapter 31 in [16] and section 5 in [2]. It seems to us that only Gawędzki's work [7] succeeded in giving a rigorous meaning to the path integral in phase space. His approach is similar to Ito's one in [11] where the path integral in configuration space was studied. The assumptions put on $H(t)$ in [7] will be mentioned later in this section.

In the present paper we study time-slicing approximation of the Feynman path integral in phase space and prove its convergence under some general assumptions. Paths

*Research partially supported by Grant-in-Aid for Scientific No. 10640176 and No. 13640161, Ministry of Education, Science, and Culture, Japanese Government.