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# ORBITS OF HERMANN ACTIONS

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## 1. Introduction

In this paper we consider the parallel translations of the normal bundles of the orbits of Hermann actions on compact symmetric spaces and represent such parallel translations by the group actions (Theorem 2.1). Using this we can show that their mean curvature vectors are parallel (Corollary 2.8), moreover those of hyperpolar actions are parallel (Corollary 2.13).

We first review some definitons and previous results concerning isometric group actions on compact symmetric spaces. Let $(G, K_1)$ and $(G, K_2)$ be compact symmetric pairs. Then $K_2$ acts isometrically on $G/K_1$, which is a compact symmetric space. This action of $K_2$ on $G/K_1$ is called a *Hermann action*.

The Hermann actions are examples of hyperpolar actions, which is defined in the following. Let $G$ be a Lie group acting isometrically on a Riemannian manifold $M$. A closed submanifold $\Sigma$ of $M$ is called a *section*, if all orbits of the action of $G$ meet $\Sigma$ perpendicularly. The action of $G$ on $M$ is said to be *hyperpolar*, if there exists a section which is flat with respect to the induced Riemannian metric. The codimension of the orbit of highest dimension is called the *cohomogeneity*. The isometric actions on compact symmetric spaces of cohomogeneity one are another examples of hyperpolar actions. Recently Kollross [8] proved that the hyperpolar actions on compact symmetric spaces are Hermann actions or cohomogeneity one actions.

We next review previous results concerning geometry of orbits of isometric group actions on symmetric spaces. The linear isotropy representations of symmetric pairs have sections which are maximal Abelian subspaces, so they are hyperpolar actions. All of their orbits have parallel mean curvature vectors, which was proved by Kitagawa-Ohnita [6]. Ohnita [9] considered the parallel translations of the normal bundles of the orbits of the linear isotropy actions on compact symmetric spaces and represent such parallel translations by the group actions. One can prove the result of Kitagawa-Ohnita mentioned above by this. Heintze-Olmos [1] also considered such parallel translations and described the normal holonomy groups of the orbits. For compact symmetric space $G/K$, Hirohashi-Song-Takagi-Tasaki [4] and Hirohashi-Ikawa-Tasaki [3] considered some geometric properties of orbits of the linear isotropy action on $T_o(G/K)$ and the isotropy action on $G/K$.