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# SYMMETRY OF THE TANGENTIAL CAUCHY-RIEMANN EQUATIONS AND SCALAR CR INVARIANTS

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## Introduction

Let $\Omega = \{r > 0\}$ be a bounded strictly pseudoconvex domain in $\mathbb{C}^{n+1}$ with smooth ($C^\infty$) boundary and let $K_\Omega$ be the Bergman kernel defined on $\Omega$. In [3], C. Fefferman proved

$$K_\Omega(Z, Z) = \frac{\phi_\Omega}{r^{n+2}} + \psi_\Omega \ln r,$$

where $\phi_\Omega$ and $\psi_\Omega$ are functions that are $C^\infty$ up to $\partial\Omega$.

$\partial\Omega$ inherits a geometric structure, called CR structure, from $\mathbb{C}^{n+1}$ which is relevant for the biholomorphic equivalence of $\Omega$. Fefferman's program, initiated in [5], is to compute all the scalar CR invariants of $\partial\Omega$ and to express the asymptotic expansion of $\phi_\Omega$ modulo $O(r^{n+2})$ and $\psi_\Omega$ modulo $O(r^\infty)$ in terms of scalar CR invariants of $\partial\Omega$.

Fefferman's invariant theory was developed further by T.N. Bailey, M.G. Eastwood, C.R. Graham, G. Komatsu and K. Hirachi, see [1], [7] and [8]. The main method is to obtain a defining function which is invariant under biholomorphic maps up to a power of determinants of biholomorphic maps and to construct a Kähler-Lorentz metric on a line bundle of $\Omega$ which is invariant under local biholomorphic maps and unique modulo $O(r^{n+1})$.

In present paper our approach is viewing the CR invariants of a real hypersurface $M$ of $\mathbb{C}^{n+1}$ as a scalar function defined on the jet space of CR embedding $F : M \to \mathbb{C}^{n+1}$ which is invariant under deformation of embedding. We express necessary and sufficient condition for scalar CR invariants using symmetry of the tangential Cauchy-Riemann equations.

Let $M = \{r = 0\}$ be a $C^\infty$ real hypersurface in $\mathbb{C}^{n+1}$ and let $\{L_j\}_{j=1,\dots,n}$ be a $C^\infty$ basis of the CR structure bundle $H^{1,0}(M) = \mathbb{C}T(M) \cap T^{1,0}(\mathbb{C}^{n+1})$. A mapping $F = (f^1, \dots, f^{n+1}) : M \to \mathbb{C}^{n+1}$ is a CR embedding if

$$\overline{L}_j f^k = 0,\ j = 1, \dots, n,\ k = 1, \dots, n+1 \tag{0.1}$$

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