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GROWTH PROPERTIES OF HYPERPLANE INTEGRALS OF SOBOLEV FUNCTIONS IN A HALF SPACE

Dedicated to Professor Masayuki Ito on the occasion of his sixtieth birthday

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1. Introduction

Let $\mathbf{D} \subset \mathbf{R}^n$ $(n \geq 2)$ denote the half space

$$\mathbf{D} = \{x = (x', x_n) \in \mathbf{R}^{n-1} \times \mathbf{R}^1 : x_n > 0\}$$

and set

$$\mathbf{S} = \partial \mathbf{D};$$

we sometimes identify $x' \in \mathbf{R}^{n-1}$ with $(x', 0) \in \mathbf{S}$. We define the hyperplane integral $S_q(u)$ over $\mathbf{S}$ by

$$S_q(u) = \left(\int_{\mathbf{S}} |u(x')|^q dx' \right)^{1/q}$$

for a measurable function u on $\mathbf{S}$ and $q > 0$.

Set

$$U_r(x') = u(x', r) - \sum_{k=0}^{m-1} \frac{r^k}{k!} \left[\left(\frac{\partial}{\partial x_n} \right)^k u \right] (x', 0)$$

for quasicontinuous Sobolev functions u on $\mathbf{D}$, where the vertical limits

$$\left(\frac{\partial}{\partial x_n} \right)^k u(x', 0) = \lim_{x_n \to 0} \left(\frac{\partial}{\partial x_n} \right)^k u(x', x_n)$$

exist for almost every $x' = (x', 0) \in \partial \mathbf{D}$ and $0 \leq k \leq m - 1$ (see [8, Theorem 2.4, Chapter 8]).

Our main aim in this note is to study the existence of limits of $S_q(U_r)$ at $r = 0$. More precisely, we show (in Theorem 3.1 below) that

$$\lim_{r \to 0} r^{-\omega} S_q(U_r) = 0$$