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# RADIAL VISCOSITY SOLUTIONS OF THE DIRICHLET PROBLEM FOR SEMILINEAR DEGENERATE ELLIPTIC EQUATIONS

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## 1. Introduction

In this paper we are concerned with the Dirichlet problem (hereafter called (DP)) for the semilinear degenerate elliptic equation:

$$\mathcal{F}[u](x) := -g(|x|)\Delta u + f(|x|, u) = 0 \quad \text{in} \quad B_R \tag{1.1}$$

$$u = \beta \quad \text{on} \quad \partial B_R, \tag{1.2}$$

where $B_R = \{x \in \mathbf{R}^N; |x| < R\}$, $N \geq 2$, $g : [0, R] \to R^+ = [0, \infty)$ is a given continuous function, $\Delta$ is the Laplacian, and $\beta$ is a given real number.

In 1981 symmetry properties of positive solutions of nonlinear elliptic equations were investigated by Gidas, Ni and Nirenberg [10, 11]. Since then there have been lots of works published in this direction (See, e.g. [1], [2], [9] and [14, 15]). There are also many works on the boundary value problem for elliptic equations whose coefficients are singular at the origin or on the boundary. See, e.g. Ebihara and Furusho [7] and Senba, Ebihara and Furusho [17], H. Egnell [8], Conti,Crotti and Pardo [3]. It is natural to ask whether solutions are radially symmetric for degenerate elliptic equations such as our (1.1). That is, we want to study the case when the degeneracy of equations arises in the interior of domains. In this case it is well known that in general we can not expect to have $C^2$-solutions.

In recent years, a considerable number of works have been done on the theory and applications of viscosity solutions. We refer the reader to the Monograph by Crandall, Ishii and Lions [4] for definitions, details and references. As regards earlier related works, in 1992, Crandall and Huan [5] studied the existence, uniqueness and non-uniqueness of viscosity solutions of the two-point boundary value problem for the linear ordinary differential equation:

$$\begin{cases} -a(x)u'' + c(x)u = f(x) & \text{in} \quad (-1, 1) \\ u(-1) = \gamma_-, \quad u(1) = \gamma_+ \end{cases}$$

under the assumption that $a, c, f \in C[-1, 1]$, $a \geq 0$ and $c \geq 0$. Soon after, Tomita