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BOUND STATES OF PERTURBED HAMILTONIANS IN THE STRONG COUPLING LIMIT

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1. Introduction

Let $V \in L^2_{\mathrm{loc}}(\mathbb{R}^d)$ be a non negative potential; the now classical Cwikel-Lieb-Rozenblum (CLR) inequality asserts that the number $N(V)$ of negative eigenvalues for the Schrödinger operator $-\Delta - V$ verifies

$$N(V) \leq C_d \int V(x)^{d/2} dx,$$

when $d > 2$. More generally Egorov [7] proved that if the Laplacian is replaced by an elliptic positive differential operator of order $2l$, the CLR inequality remains valid when $d > 2l$ with $d/2$ replaced by $d/(2l)$. Recently Rozenblum and Solomiak [13] obtained a general result for a wide class of perturbed hamiltonians $H_0 - V$.
When the RHS in the CLR inequality is finite we can define the number $N(\lambda V)$ of negative eigenvalues for $H_0 - \lambda V$. The principal term of the asymptotic for $N(\lambda V)$ when $\lambda \to +\infty$ was found independently by Birman and Borzov [3] and Martin [10] in the case $d \geq 3$:

$$N(\lambda V) \sim \lambda^{d/2} \int V(x)^{d/2} dx \ .$$

Tamura [15] obtained later on a sharp remainder estimate for $d = 3$:

$$N(\lambda V) = \lambda^{3/2} \int V(x)^{3/2} dx + O(\lambda), \ \text{ when } \lambda \to +\infty,$$

for very regular potential like $\langle x \rangle^{-m}$ with $m > 2$. More recently Birman and Solomiak [4] obtained asymptotics for $H_0 = (-\Delta)^l$ and $d > 2l$ (and also for $d < 2l$ but we do not consider this case here).

The aim of this paper is to obtain similar Weyl asymptotics when H_0 is a pseudodifferential operator. Global ellipticity is not required; roughly speaking, we suppose that the complete symbol of H_0 is hypoelliptic and satisfy a condition which looks like ellipticity near $\xi = 0$, see the complete assumptions in 2.1. A typical example is for instance the relativistic Laplacian $H_0 = (-\Delta + 1)^{1/2} - 1$. The class of potentials V considered here is less restrictive than those considered in [15].