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LIMITING ABSORPTION PRINCIPLE FOR DIRAC OPERATOR WITH CONSTANT MAGNETIC FIELD AND LONG-RANGE POTENTIAL

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1. Introduction

The Dirac Hamiltonian with magnetic vector potential $\mathbf{a} = (a_j(x))_{j=1,\dots,d}$ is expressed by the following form

$$H(\mathbf{a}) = \sum_{j=1}^{d} \gamma_j(P_j - a_j) + m\gamma_{d+1} + V, \tag{1.1}$$

where $P_j = 1/i\partial_{x_j}$, V is a multiplication of an Hermitian matrix $V(x)$. m is the mass of electron. The matrices $\{\gamma_j\}$ satisfy the following relations

$$\gamma_j\gamma_k + \gamma_k\gamma_j = 2\delta_{jk}\mathbf{1} \quad (j, k = 1, \dots, d+1). \tag{1.2}$$

Here δ_{jk} is Kronecker's delta and $\mathbf{1}$ is an identity matrix. We assume that the speed of the light $c = 1$. When $V \equiv 0$, the square of $H(\mathbf{a})$ has the form

$$H(\mathbf{a})^2 = \sum_{j=1}^{d}(P_j - a_j)^2 + m^2 + \frac{1}{i}\sum_{1\le j<k\le d} b_{jk}(x)\gamma_j\gamma_k, \tag{1.3}$$

where

$$b_{jk}(x) = \partial_{x_k}a_j(x) - \partial_{x_j}a_k(x). \tag{1.4}$$

It is called Pauli's Hamiltonian. The skew symmetric matrix $(b_{jk}(x))$ is the magnetic field associated with $\mathbf{a}$. We say the magnetic field is asymptotically constant if it satisfies the following conditions as $|x| \to \infty$:

$$b_{jk}(x) \to {}^{\exists}\Lambda_{jk} \quad (1 \le j, k \le d), \tag{1.5}$$

where $(\Lambda_{jk})_{j,k}$ is a constant matrix.

The aim of this paper is to prove the limiting absorption principle for $H(\mathbf{a})$ with a constant magnetic field $(b_{jk}(x))$ and a long-range electric potential $V(x)$ when $d = 3$.