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MODULI OF ALGEBRAIC SL_3-VECTOR BUNDLES OVER ADJOINT REPRESENTATION

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1. Introduction and result

Let G be a reductive complex algebraic group and P a complex G-module. We consider algebraic G-vector bundles over P. An algebraic G-vector bundle E over P is an algebraic vector bundle $p : E \to P$ together with a G-action such that the projection p is G-equivariant and the action on the fibers is linear. We assume that G is non-abelian since every G-vector bundle over P is isomorphic to a trivial G-bundle $P \times Q \to P$ for a G-module Q when G is abelian by Masuda-Moser-Petrie [12]. We denote by $\mathrm{VEC}_G(P, Q)$ the set of equivariant isomorphism classes of algebraic G-vector bundles over P whose fiber over the origin is a G-module Q. The isomorphism class of a G-vector bundle E is denoted by $[E]$. The set $\mathrm{VEC}_G(P, Q)$ is a pointed set with a distinguished class $[\mathbf{Q}]$ where $\mathbf{Q}$ is the trivial G-bundle $P \times Q$, and can be non-trivial when the dimension of the algebraic quotient space $P/\!/G$ is greater than 0 ([15], [2], [13], [11]). In fact, Schwarz ([15], cf. Kraft-Schwarz [5]) showed that $\mathrm{VEC}_G(P, Q)$ is isomorphic to an additive group $\mathbb{C}^p$ for a nonnegative integer p determined by P and Q when $\dim P/\!/G = 1$. When $\dim P/\!/G \geq 2$, $\mathrm{VEC}_G(P, Q)$ is not necessarily finite-dimensional. In fact, $\mathrm{VEC}_G(P \oplus \mathbb{C}^m, Q) \cong (\mathbb{C}[y_1, \cdots, y_m])^p$ for a G-module P with one-dimensional quotient [9]. Furthermore, Mederer [14] showed that $\mathrm{VEC}_G(P, Q)$ can contain a space of uncountably-infinite dimension. He considered the case where G is a dihedral group $D_m = \mathbb{Z}/2\mathbb{Z} \ltimes \mathbb{Z}/m\mathbb{Z}$ and P is a two-dimensional G-module V_p, on which $\mathbb{Z}/m\mathbb{Z}$ acts with weights p and $-p$ and the generator of $\mathbb{Z}/2\mathbb{Z}$ acts by interchanging the weight spaces. Mederer showed that $\mathrm{VEC}_{D_3}(V_1, V_1)$ is isomorphic to $\Omega^1_{\mathbb{C}}$ which is the universal Kähler differential module of $\mathbb{C}$ over $\mathbb{Q}$. In this article, we show that under some conditions there exists a surjection from $\mathrm{VEC}_G(P, Q)$ to $\mathrm{VEC}_{D_3}(V_1, V_1) \cong \Omega^1_{\mathbb{C}}$. It is induced by taking a H-fixed point set E^H for $[E] \in \mathrm{VEC}_G(P, Q)$ where H is a reductive subgroup of G (cf. Proposition 2.3). In particular, we obtain the first example of a moduli space of uncountably-infinite dimension for a connected group.

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