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ON AUSLANDER-REITEN COMPONENTS AND PROJECTIVE LATTICES OF p-GROUPS

Dedicated to Professor Yukio Tsushima on his 60th birthday

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Introduction

Let G be a finite group, p a prime number which divides the order of G, and $(K, \mathcal{O}, k)$ a p-modular system, i.e., $\mathcal{O}$ is a complete discrete valuation ring of characteristic zero with maximal ideal (π), $k(:= \mathcal{O}/(\pi))$ is the residue field of $\mathcal{O}$ of characteristic $p > 0$, and K is the field of fractions of $\mathcal{O}$. R is used to denote either $\mathcal{O}$ or k. All the RG-modules considered here are R-free and finitely generated over R.

Let $\Gamma(RG)$ be the Auslander-Reiten quiver of RG. For a connected component Θ of $\Gamma(RG)$, we denote by Θ_s the stable part of Θ obtained from Θ by removing all projective RG-modules and arrows attached to them. In [16], P. J. Webb showed that the tree class of Θ_s is either a Euclidean diagram or one of the infinite trees A_∞, B_∞, C_∞, D_∞ and A_∞^∞ if the modules in Θ do not lie in a block of cyclic defect.

It was shown in [10] that if G is a p-group and $\mathcal{O}G$ is of infinite representation type, and furthermore if $(\pi) \supsetneqq (2)$ in the case where $p = 2$ and G is the Klein four group, then the stable part of the connected component of $\Gamma(\mathcal{O}G)$ containing the trivial $\mathcal{O}G$-lattice $\mathcal{O}_G$ has tree class A_∞. The purpose of this paper is to show the following.

Theorem. *Let G be a p-group and Δ the connected component of $\Gamma(\mathcal{O}G)$ containing the projective $\mathcal{O}G$-lattice $\mathcal{O}G$. Suppose that $\mathcal{O}G$ is of infinite representation type. Suppose further that $(\pi) \supsetneqq (2)$ in the case where $p = 2$ and G is the Klein four group. Then the tree class of the stable part Δ_s of Δ is A_∞.*

It is known that the group ring $\mathcal{O}G$ of a finite p-group G is of finite representation type if and only if one of the following cases arises: (i) $G = C_2$; (ii) $G = C_3$ and $(3) \supseteq (\pi^3)$; (iii) $G = C_p$ and $(p) \supseteq (\pi^2)$; (iv) $G = C_{p^2}$ and $(p) = (\pi)$, where C_{p^n} is the cyclic group of order p^n. See [4]. Also, it is known that if G is the Klein four group and $(\pi) = (2)$, then the tree class of the stable part of the connected component of $\Gamma(\mathcal{O}G)$ containing the projective $\mathcal{O}G$-lattice $\mathcal{O}G$ is $\tilde{D}_4$ (Proposition 3.4 of [5]).

In the rest of this paper G will always be a finite p-group. In Sections 1, we con-