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ON PERIODIC β-EXPANSIONS OF PISOT NUMBERS AND RAUZY FRACTALS

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0. Introduction

Let λ be the real maximum solution of the polynomial $p(x)$: $k_1, k_2 \in \mathbf{N}$ and $k_1 \geq k_2$ $(k_1 \neq 0)$

$$p(x) = x^3 - k_1 x^2 - k_2 x - 1.$$

The polynomial $p(x)$ is given as the characteristic polynomial of the matrix M:

$$M = \begin{bmatrix} k_1 & k_2 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

And for each k_1, k_2 the real cubic number λ is a Pisot number. A Pisot number is an algebraic integer whose conjugates other than itself have modulus less than one. Hence,

$$|\lambda'|, |\lambda''| < 1,$$

where λ', λ'' are algebraic conjugates of λ. We denote the column and row eigenvectors of λ by

$$M \begin{pmatrix} 1 \\ \alpha \\ \beta \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ \alpha \\ \beta \end{pmatrix} \quad \text{and} \quad {}^t M \begin{pmatrix} 1 \\ \gamma \\ \delta \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ \gamma \\ \delta \end{pmatrix},$$

where t indicates the transpose.
Let $T_\lambda : [0, 1) \to [0, 1)$ be the transformation given by

$$T_\lambda x = \lambda x - [\lambda x],$$