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EQUIVARIANT K-THEORETIC EULER CLASSES AND MAPS OF REPRESENTATION SPHERES

Dedicated to Professor Fuichi Uchida on his 60th birthday

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0. Introduction

Let G be a compact Lie group, and U a unitary representation of G. The unit sphere SU of U is called a *representation sphere* of G. In this paper we study G-maps between finite dimensional representation spheres of G.

In Atiyah and Tall [2], Bartsch [3], tom Dieck and Petrie [4], Komiya [5], [6], Liulevicious [7], and Marzantowicz [8], the equivariant K-theory is successfully employed for the study of G-maps. In [2] and [4], that is employed for the study of degrees of G-maps between representation spheres. In [3], [5], [6] and [7], that is employed to obtain necessary conditions for the existence of G-maps. In [8], the equivariant K-theoretic Lefschetz number is defined.

The main tool in this paper is also the equivariant K-theory. We give the definitions of the Thom class $tU \in K_G(U)$ of U and the Euler class $eU \in K_G(\mathrm{pt}) = R(G)$, and then show that if there exists a G-map $f : SU \to SW$ between representation spheres SU and SW then $eW = z(f) \cdot eU$ for some element $z(f) \in R(G)$ (Theorem 1.2). Using this equality, we show that if G is connected then the degree of f is uniquely determined only by U and W (Theorem 4.1).

If G is compact abelian, then the degree of f is more explicitly discussed. Let $S^1 = \{z \in \mathbb{C} \mid |z| = 1\}$ be the circle group of complex numbers with absolute value 1, and $\mathbb{Z}_n$ be the cyclic group of order n considered as a subgroup of S^1. For any integer i, let $V_i = \mathbb{C}$ be a complex representation of S^1 and $\mathbb{Z}_n$ given by $(z, v) \mapsto z^i v$ for $z \in S^1$ (or $\mathbb{Z}_n$) and $v \in V_i$. A compact abelian group G decomposes into a cartesian product

$$G = T^k \times \mathbb{Z}_{n_1} \times \cdots \times \mathbb{Z}_{n_l},$$

where $T^k = S^1 \times \cdots \times S^1$, the cartesian product of k copies of S^1. Letting γ be a sequence $(a_1, \ldots, a_k, b_1, \ldots, b_l)$ of integers, denote by V_γ the tensor product

$$V_{a_1} \otimes \cdots \otimes V_{a_k} \otimes V_{b_1} \otimes \cdots \otimes V_{b_l},$$