THE GENUS FIELD AND GENUS NUMBER IN ALGEBRAIC NUMBER FIELDS

YOSHIOMI FURUTA[1]

Dedicated Professor KIYOSHI NOSHIRO on his 60th birthday

Let k be an algebraic number field and K be its normal extension of finite degree. Then the *genus field* K^* *of* K *over* k is defined as the maximal unramified extension of K which is obtained from K by composing an abelian extension over k[2]. We call the degree $(K^* : K)$ the *genus number of* K *over* k.

In the case where k is the rational number field, the genus number is studied by Hasse [2] for quadratic extensions, by Iyanaga and Tamagawa [3] and by Leopoldt [6] for abelian extensions, and by Fröhlich [1], [1′] for normal extensions.

At the present time, there is no difficulty to treat the genus number in general, for which, however, no convenient literature is available. So, in this rather expository paper, we shall give a general formula for the genus number, which would have some meaning especially in the investigation of the class number relation[3].

1. For any finite or infinite prime $\mathfrak{p}$ of k we denote by $k_{\mathfrak{p}}$ the $\mathfrak{p}$-completion of k ; $U_{\mathfrak{p}}$ the unit group of $k_{\mathfrak{p}}$; J_k the idele group of k, into which we embed $k^\times$ and $k_{\mathfrak{p}}^\times$ in usual way[4]; and $U_k = \prod_{\mathfrak{p}} U_{\mathfrak{p}}$ the unit idele group of k.

A subgroup H of J_k is called *admissible* if H is a closed subgroup of finite index in J_k and contains $k^\times$. Then an admissible subgroup of J_k and an abelian extension over k of finite degree correspond to each other by the class field theory.

For an Galois extension K/k we denote by $N_{K/k}$ the norm from K to k and

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2) For normal extensions K/k this definition is according to Fröhlich [1].

3) Cf. Fröhlich [1], Iwasawa [4], Kuroda [5] and Yokoyama [7].

4) We mean by $k^\times$ the multiplicative group of all non-zero elements of k.