ON BLOCK IDEMPOTENTS OF MODULAR GROUP RINGS

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To the memory of TADASI NAKAYAMA

We consider a group G of finite order $g=p^a g'$, where p is a prime number and $(p, g')=1$. Let Ω be the algebraic number field which contains the g-th roots of unity. Let $K_1, K_2, \ldots, K_n$ be the classes of conjugate elements in G and the first $m(\leqq n)$ classes be p-regular. There exist n distinct (absolutely) irreducible characters $\chi_1, \chi_2, \ldots, \chi_n$ of G. Let $\mathfrak{o}$ be the ring of all algebraic integers of Ω and let $\mathfrak{p}$ be a prime ideal of $\mathfrak{o}$ dividing p. If we denote by $\mathfrak{o}^*$ the ring of all $\mathfrak{p}$-integers of Ω, then $\mathfrak{p}$ generates an ideal $\mathfrak{p}^*$ of $\mathfrak{o}^*$ and we have

$$\Omega^* = \mathfrak{o}^*/\mathfrak{p}^* \cong \mathfrak{o}/\mathfrak{p}$$

for the residue class field. The residue class map of $\mathfrak{o}^*$ onto Ω^* will be denoted by an asterisk; $\alpha \to \alpha^*$.

Let $\Gamma = \Gamma(G)$ be the modular group ring of G over Ω^* and let

$$Z = Z_1 \oplus Z_2 \oplus \cdots \oplus Z_s$$

be the decomposition of the center $Z=Z(G)$ of Γ into indecomposable ideals Z_σ. Then the ordinary irreducible characters χ_i and the modular irreducible characters φ_κ of G (for p) are distributed into s blocks $B_1, B_2, \ldots, B_s$, each χ_i and φ_κ belonging to exactly one block B_σ. We determined in [6] explicitly the primitive orthogonal idempotents δ_σ of Z corresponding to B_σ in the following way. We set

$$b_\alpha = \sum_{\chi_i \in B_\sigma} z_i \chi_i(a_\alpha^{-1})/g \qquad (a_\alpha \in K_\alpha)$$

where $z_i = \chi_i(1)$. Let U_κ be the indecomposable constituent of the regular representation of G corresponding to the modular irreducible representation F_κ and denote by u_κ its degree. We see that $b_\alpha = \sum_{\varphi_\kappa \in B_\sigma} u_\kappa \varphi_\kappa(a_\alpha^{-1})/g \in \mathfrak{o}^*$ for p-regular

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