ON SOME BOUNDARY PROBLEMS IN THE THEORY OF CONFORMAL MAPPINGS OF JORDAN DOMAINS

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1. It is a well-known result in the theory of conformal mappings of Jordan domains that if a domain D in the z-plane bounded by a closed Jordan curve C is mapped conformally on the disc $|w|<1$ by a function $w=f(z)$, analytic and univalent in D, then $f(z)$ will be continuous on the closure of D and will map C on $|w|=1$ in a one to one manner (Carathéodory [2]), and that if C is rectifiable, then $f(z)$ will map sets E of points of linear measure zero on C onto sets of linear measure zero on the circumference $|w|=1$ and sets E of positive linear measure onto sets of positive linear measure on $|w|=1$ (F. and M. Riesz [12] and Lusin and Privaloff [8]). If the condition that C is rectifiable is dropped, however, the above metric property can no longer be asserted for $f(z)$ on C. In fact, Lavrentieff gives in his paper [5] an example of a domain D bounded by a non-rectifiable closed Jordan curve C, by the conformal map $w=f(z)$ of which on the unit disc $|w|<1$ a set E of linear measure zero on C is mapped onto a set of positive linear measure on $|w|=1$ and Lohwater and Seidel [6] and Lohwater and Piranian [7] show that there exist Jordan domains D, by the conformal map $w=f(z)$ of which on $|w|<1$ a set E of positive linear or two-dimensional measure on C is mapped onto a set of linear measure zero on $|w|=1$. R. Nevanlinna [10; p. 107] also states without proof that an example of a set E can be given which belongs to the boundaries of two Jordan domains D_1 and D_2 and is mapped onto a set of linear measure zero by the conformal map of D_1 on the unit disc, while it is mapped onto a set of positive linear measure by the map of D_2 on the unit disc. Here we raise the following problems:

(i) Under what metrical condition for E can the condition that C is rectifiable be dropped to assert that it is mapped onto a set of linear measure zero?

(ii) Under what metrical condition for E can the condition that C is recti-

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