ON CARTAN PSEUDO GROUPS

A. M. RODRIGUES

Let M be a domain in an Euclidean space and let Γ be a pseudo group of transformations* of M. We say that Γ is a Cartan pseudo group [1, 2] if the following conditions are satisfied:

1) There exists a domain M' and a projection $\rho : M \rightarrow M'$, such that the orbits of Γ are the fibers of the projection ρ. We assume moreover that there is a system of coordinates (x) of M' and a system of coordinates (x, y) of M such that the fibers of ρ are defined by $(x) = \text{constants}$,

2) There are forms ω^i, $\tilde{\omega}^\lambda$, $i = 1 \cdots m$, $\lambda = 1 \cdots n$, defined in D such that

a) $(\omega^i, \tilde{\omega}^\lambda)$ is a basis of the space of linear forms at every point of M,

$$\text{(1)} \qquad \text{b) } d\omega^i = \frac{1}{2} c^i_{jk} \omega^j \wedge \omega^k + a^i_{j\lambda} \omega^j \wedge \tilde{\omega}^\lambda$$

where c^i_{jk}, $a^i_{j\lambda}$ are functions on M which depend on (x) only,

c) $\omega^r = dx^r$ for $1 \leq r \leq \text{dimension } M'$,

d) The matrices $a_\lambda = \|a^i_{j\lambda}\|$ are linearly independent at every point of M,

e) Let π_1 and π_2 be respectively the projections of $M \times M$ into the first and second factors. The closed differential system Σ on $M \times M$, with independent variables $x \circ \pi_1$, $y \circ \pi_1$ generated by

$$x^r \circ \pi_1 - x^r \circ \pi_2 = 0, \; 1 \leq r \leq \text{dimension } M',$$
$$\pi_1^* \omega^i - \pi_2^* \omega^i = 0 \qquad 1 \leq i \leq m$$

is in involution at every integral point,

3) A local transformation f of M is in Γ if and only if f preserves the forms ω^i, i.e. $f^* \omega^i = \omega^i$, $i = 1, \ldots, m$.

In this note we prove that every differential form on M which is invariant under all transformations of a Cartan pseudo group Γ is a linear combination of the forms ω^i the coefficient being functions of x only.

Received on 1 May, 1962.

* All maps and differential forms considered in this note are assumed to be analytic.