GROUP CHARACTERS AND NORMAL HALL SUBGROUPS

P. X. GALLAGHER

To RICHARD BRAUER on his 60th birthday

1. Introduction

Let G be a finite group and let ψ be an (ordinary) irreducible character of a normal subgroup N. If ψ extends to a character of G then ψ is invariant under G, but the converse is false. In section 3 it is shown that if ψ extends coherently to the intermediate groups H for which H/N is elementary, then ψ extends to G. If N is a Hall subgroup, then in order for ψ to extend to G it is sufficient that ψ be invariant under G. This leads to a construction of the characters of G from the characters of N and the characters of the subgroups of G/N in this case.

Let $1(H)_G$ denote the character of G induced by the 1-character of the π-Hall subgroup H. If H is normal in G then the degree of each irreducible component of $1(H)_G$ divides the index of H. In section 4, the converse is proved in the case in which G is π-solvable and in the case in which G has a nilpotent π'-Hall subgroup.

2. Notation and Preliminary Results

In what follows, group means finite group. For a character ψ of a subgroup H of a group G, ψ_G denotes the character of G induced by ψ (for a definition, see [1]). Induction has the following properties:

(i) If $H \subset K \subset G$, then $(\psi_K)_G = \psi_G$.

(ii) If χ is a character of G, then $(\psi_G, \chi)_G = (\psi, \chi)_H$.

If χ_1 and χ_2 are characters of G, then $\chi_2 \in \chi_1$ means that there is a character χ_3 such that $\chi_1 = \chi_2 + \chi_3$. For example, $\psi \in \psi_G | H$ for each character ψ of the subgroup H of G.

The 1-character of G is denoted by $1(G)$.

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