ON TRANSITIVE SIMPLE GROUPS OF DEGREE $3p$*)

To RICHARD BRAUER on his sixtieth birthday

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Let Ω be the set of symbols $1, 2, \ldots, 3p$, where p is an prime number greater than 3. Let $\mathfrak{G}$ be a transitive permutation group on Ω, which is simple and in which the normalizer of a Sylow p-subgroup has order $2p$. Our purpose is to prove the following two theorems:

THEOREM 1. *If $\mathfrak{G}$ is primitive on Ω, then $p=5$ and $\mathfrak{G}$ is isomorphic to the alternating group $\mathfrak{A}_6$ of degree* 6.

THEOREM 2. *If $\mathfrak{G}$ is imprimitive on Ω, then $\mathfrak{G}$ is isomorphic to the linear fractional group $LF(2, 2^m)$ with $2^m+1=p$.*

Our proof of Theorem 1 is fairly complicated. Theorem 1 implies that such a group $\mathfrak{G}$ cannot be doubly transitive. This fact will be proved in § 2. There the irreducible characters of dimension two of the symmetric group on Ω play an essential role as in our previous papers [14], [15]. We need also, however, recent result of Thompson [18] concerning groups of odd order. In § 3 we treat, roughly speaking, the almost doubly transitive case. There a result of Wielandt concerning the eigenvalues of intertwining matrices is very useful [21]. With the help of this theorem of Wielandt, some results of Brauer and Suzuki [4], [17] concerning groups whose Sylow 2-subgroups are dihedral groups of order either 4 or 8 respectively can be used. In § 4 we consider, roughly speaking, the strongly simply transitive case. For this case we need again some deep results.

Theorem II is a simple consequence of our previous result [14].

Finally, we want to emphasize that we need from beginning to end Brauer's p-block theory of irreducible characters.

Reseived September 5, 1961.

*) This work was supported by the United States Army under Contract No. DA-ARO(D)-31-124-G 86 monitored by the Army Research Office.