A MAXIMAL RIEMANN SURFACE

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We let the notations be as in [3]. Then, in the category $\mathfrak{C}$ of all bordered Riemann surfaces, the following inclusion diagram holds [3, Theorem 9]:

$$M_0 \subset O_{HB} \begin{matrix} \subset O_{HD} \subset \\ \subset O_{KB} \subset \end{matrix} O_{KD} \subset M_2.$$

Further, from a theorem of Kuramochi [4] (see also Constantinescu and Cornea [2]), it easily follows that the class O_{AD} is not contained in M_2. On the other hand, it is well known that M_2(which equals O_{SB} for ordinary planar surfaces) is not contained in O_{AD} (see Ahlfors and Beurling [1]).

Now let $\mathfrak{C}_0$ be the subcategory of bordered Riemann surfaces without planar ideal boundary. Then $\mathfrak{C}_0 \cap M_2 = M =$ the class of all maximal bordered Riemann surfaces. Hence the question whether M is or not contained in O_{AD} naturally arises; it was first considered by Sario [5]. This note contains the negative answer to Sario's question.

Let $X = R \cup B$ and $X_0 = R_0 \cup B_0$ be two bordered Riemann surfaces. We recall that a continuous map $f: X \to X_0$ is said to be ***distinguished*** if $f(B) \subset B_0$, and ***proper*** if, for any compact $K_0 \subset X_0$, $f^{-1}(K_0)$ is compact. Let M_1 be the class of all bordered Riemann surfaces with absolutely disconnected ideal boundary.

THEOREM 1. *Suppose there exists a distinguished proper conformal map $f: X \to X_0$. Then $X \in M_1$ if and only if $X_0 \in M_1$.*

Proof. Let β and β_0 be the nowhere disconnecting and 0-dimensional ideal boundaries of X and X_0. Then the spaces $X^* = X \cup \beta$ and $X_0^* = X_0 \cup \beta_0$ are compact and locally connected, and the sets β and β_0 are nowhere disconnecting and 0-dimensional. By Lemma 2 in [3], the proper map $f: X \to X_0$ can be extended to a continuous map $f^*: X^* \to X_0^*$ satisfying $f^*(\beta) = \beta_0$ and $f^{*-1}(\beta_0) = \beta$.

Received June 26, 1961.