ON SOME CRITERIA FOR A SET TO BE OF CLASS $N_{\mathfrak{B}}$

NOBUYUKI SUITA

1. Let D be a plane domain containing the point at infinity and E its complementary closed set. As to a sufficient condition for a compact set E to be of class $N_{\mathfrak{B}}$, Pfluger-Mori's criterion is well-known (Pfluger [10], Mori [6]). Various relations between the conditions of this type and the Hausdorff measure of the set E have been investigated recently by Kuroda and Ozawa (Kuroda [5], Ozawa and Kuroda [8], Ozawa [7]). For example they showed that Pfluger-Mori's condition implies that the set E is of one dimensional measure zero under some additional conditions (cf. [7], [8]). In the present paper we shall give an alternative proof of Pfluger-Mori's criterion and another criterion using analytic module and, further, prove some criteria for the set E to be of one dimensional measure zero.

2. We consider a set of doubly connected domains $R_n^{(k)}$ ($k = 1, 2, \ldots, \nu(n) < \infty$; $n = 1, 2, \ldots$) satisfying the following conditions;

(i) the closure of $R_n^{(k)}$ is contained in D,

(ii) the boundary of $R_n^{(k)}$ consists of two rectifiable closed Jordan curves $C_{1n}^{(k)}$ and $C_{2n}^{(k)}$,

(iii) $C_{1n}^{(k)}$ contains $C_{2n}^{(k)}$ in its interior and the point at infinity in its exterior $F_n^{(k)}$,

(iv) the interior $G_n^{(k)}$ of $C_{2n}^{(k)}$ contains at least one point of E and the set E is contained in $\bigcup_{k=1}^{\nu(n)} G_n^{(k)}$,

(v) $R_n^{(j)}$ lies in $F_n^{(k)}$ for any $k \neq j$,

(vi) each $R_{n+1}^{(k)}$ is contained in a certain $G_n^{(k)}$ and

(vii) $\{D_n\}$ is an exhaustion of D, where D_n is defined by $\bigcap_{k=1}^{\nu(n)} (F_n^{(k)} \cup R_n^{(k)})$.

Let $\log \mu_n^{(k)}$ be the modulus of the ring domain $R_n^{(k)}$ and $\mu_n = \operatorname*{Min}_{1 \leq k \leq \nu(n)} \mu_n^{(k)}$.

Pfluger-Mori's criterion can be stated as follows.

THEOREM 1. *If there exists an exhaustion $\{D_n\}$ of D satisfying*

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