A THEOREM ON AN ANALYTIC MAPPING OF RIEMANN SURFACES

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Recently S. S. Chern [1] intended an aproach to some problems about analytic mappings of Riemann surfaces from a view-point of differential geometry. In that line we treat here orders of circular points of analytic mappings. The author expresses his thanks to Prof. K. Noshiro for his kind advices.

1. In the first we summarise formulas for conformal mappings of Riemannian manifolds (cf. [3], [4]). We take 2-dimensional Riemannian manifolds M and N of differentiable class C_2 and assume that there exists a conformal mapping f of M into N which is locally diffeomorphic. We take orthonormal frames on the tangent spaces of points of any neighborhood U of M. Then a line element ds of M can be represented as

$$ds^2 = \omega_1^2 + \omega_2^2 \tag{1}$$

with 1-forms ω_1 and ω_2. When we choose frames on the tangent spaces of $f(U)$ corresponding to those on M by the conformal mapping f, we have for a line element dt of N

$$dt^2 = \pi_1^2 + \pi_2^2 = a^2 ds^2, \tag{2}$$

where $$\pi_1 = a\omega_1, \qquad \pi_2 = a\omega_2. \qquad (a > 0) \tag{3}$$

A form ω_{12} of Riemannian connection of M is defined in U by the relations

$$d\omega_1 = \omega_2 \wedge \omega_{21}, \qquad d\omega_2 = \omega_1 \wedge \omega_{12}, \qquad \omega_{12} = -\omega_{21}. \tag{4}$$

When we put

$$da/a = b_1\omega_1 + b_2\omega_2, \tag{5}$$

a connection form $\pi_{12} = -\pi_{21}$ of Riemannian connection of N in $f(U)$ is given by

$$\pi_{12} = \omega_{12} + b_1\omega_2 - b_2\omega_1. \tag{6}$$

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