ON FROBENIUS EXTENSIONS II

TADASI NAKAYAMA and TOSIRO TSUZUKU

In Part I[1] we introduced the notion of 2. Frobenius extensions of a ring, as a generalization of Kasch's [10] Frobenius extensions and hence of classical Frobenius algebras. We proved, in I, bilinear (or sesqui-linear, rather, to follow Bourbaki's terminology) form and scalar product characterizations of Frobenius extensions in such extended sense, generalizing Kasch's and classical case, and then studied homological dimensions in them, generalizing and refining the results in Eilenberg-Nakayama [4] and Hirata [6]. Dual bases were considered in case of quasi-free (2.) Frobenius extensions Also the case of a semi-primary or S-ring ground ring was studied.

In the present Part II we continue our study of Frobenius extensions in such generalized sense. Thus we first study relative homological dimensions in them, generalizing the Maschke-Ikeda-Kasch characterization of relatively projective and relatively injective modules as well as Hirata's [6] results. Then in §7 we establish Kasch's [10] theorem on the endomorphism ring of a Frobenius ring for our generalized case. Here the removal of Kasch's S-ring assumption (which we have already discussed in our previous note [13]) and the replacement of free module property with projective or quasi-free ones make our proof more complicated, respectively in substance and in computation, than Kasch's case.

Then, in §8, we transfer to the present case the annihilator relations given in [17] for classical Frobenius algebras, on restricting ourselves to d-ideals (similar to (but slightly more general than) v-ideals in Kasch [10]). Further, in §9 we consider residue-rings of a Frobenius extension, in order to study when they are also Frobenius extensions. In these considerations Frobenius extensions are naturally taken in our generalized sense and thus deviations, some rather essential and some rather formal, from (the classical case and)

Received May 12, 1961.

[1] T. Nakayama-T. Tsuzuku, On Frobenius extensions I, Nagoya Math. J. 17 (1960), 89–110.