ON THE LOCAL THEORY OF CONTINUOUS INFINITE PSEUDO GROUPS II*

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Chapter III. Differential systems

In this chapter, we shall formulate, without proof, the theorys of exterior differential systems and of their prolongations using the language in the theory of jets developed by C. Ehresman, because such formulation seems to be most convenient in order to apply the theory to the theory of continuous infinite pseudo-groups, which we shall discuss in the next chapter. Since important theorems in our theory hold only in the real analytic case, we shall exclusively consider the real analytic case. So, we shall omit the adjective "analytic", unless explicitly stated otherwise. As for the fundamental notions in the theory of jets and of differential systems, we refer to [5] and [6], respectively. Detailed proof of the contents of this Chapter will be published in [7].

1. Differential systems and jets

Let M be a (real analytic) manifold. Denote by $\Lambda^h(M)$ the sheaf of germs of real analytic homogeneous differential forms of degree h on M. $\Lambda(M) = \sum_h \Lambda^h(M)$ is a sheaf of rings. The exterior derivative d induces a mapping $\Lambda^h(M) \to \Lambda^{h+1}(M)$. For a sheaf Φ on M and for an open set U of M denote by $\Gamma(U, \Phi)$ the set of cross-sections of Φ over U. By a differential system on M we mean a locally finitely generated subsheaf Σ of homogeneous ideals in $\Lambda(M)$ such that $d(\Sigma) \subseteq \Sigma$. By locally finitely generated, we mean the following: For any point w in M we can choose an open neighborhood V of w and $f_1, \dots, f_a \in \Gamma(V, \Sigma)$ such that each Σ_y $(y \in V)$ is generated as an ideal in $(\Lambda(M))_y$ by the germs of $f_1, \dots, f_a$ at w. Let B be a homogeneous subsheaf of $\Lambda(M)$. Then the subsheaf Ψ of ideals generated by B and dB is closed under d. Therefore, when Ψ is locally finitely generated, it is a differential system on M. If this is so, Ψ is called the differential system generated by B. A (not necessarily closed) submanifold N of M is called an integral of a differential

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