# A GENERALIZATION OF THE RING OF TRIANGULAR MATRICES

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## 1. Introduction

Let $R$ be a ring with unit, and $e$ be an idempotent in $R$ such that $(1-e)Re=0$. In this note we shall explore the relationships between homological properties of $R$ and those of its subring $eRe$.

Examples of such rings are abundant, the most common being perhaps the ring $R$ of all two-by-two upper triangular matrices over a field, where—

$$e=\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

In fact, it is easy to see that every ring of the type described above is in some sense a ring of upper triangular matrices, an observation which justifies the title of this paper.

We exhibit two applications of our results. First, we construct an example of a left semi-hereditary ring which is not right semi-hereditary, thus providing a negative answer to a question of Cartan and Eilenberg ([2], p. 15). Our second application is related to the work of Jans and Nakayama ([5]) and Nakano ([6]) on a class of semi-primary rings which is a special case of the type of ring considered here (recall that a ring $R$ is semi-primary if its Jacobson radical $N$ is nilpotent and $R/N$ satisfies minimum condition on left ideals). Our systematic treatment of the more general situation described above enables us to easily derive- and in some cases strengthen-several of the results of these authors.

Throughout this note every ring will be assumed to have a unit which acts as the identity on all modules. A ring $R$ will be called *semi-simple* if it has global dimension zero, or, equivalently, it satisfies minimum condition on left ideals and has trivial Jacobson radical ([2], p. 11). $R$ will be called *regular*

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