A GENERALIZATION OF THE ALMOST ZERO THEORY

R. SRIDHARAN*

1. Introduction

In this note we shall give a modified definition of cohomology groups for algebras. For a class of (infinite rank) algebras (which includes Frobenius algebras, group rings of infinite groups and division algebras of countable rank over fields), these groups can be characterized in a manner similar to the cohomology groups in the 'Almost Zero theory' of B. Eckmann [2]. Actually, in the case of group rings, these coincide with the cohomology groups in the almost zero theory.

We shall see that a theorem [3, Th. 11] of S. Eilenberg and T. Nakayama can be deduced from our general setting. We shall also derive a necessary and sufficient condition for modules over algebras of the above class to be weakly projective, which is a generalization of a proposition [1, p. 200] of H. Cartan and S. Eilenberg on modules over group rings. For all notions relating to Homological algebra, we refer to [1].

2. The almost zero theory

We shall recall here the definition of the almost-zero theory [1, p. 358]. Let π be a group, C an abelian group with trivial π-operators. Let X be a left π-complex. A cochain $f \in \mathrm{Hom}_Z(X_n, C)$ is called a *π-finite* cochain if for any $p \in X_n$, $f(x \cdot p) = 0$ for all but a finite number of $x \in \pi$. For any left $Z(\pi)$-module A, we have an isomorphism [1, p. 359]

$$\mathrm{Hom}_{Z(\pi)}(A, Z(\pi) \underset{Z}{\otimes} C) \approx \overline{\mathrm{Hom}}_Z(A, C)$$

where $\overline{\mathrm{Hom}}_Z(A, C)$ is the subgroup of $\mathrm{Hom}_Z(A, C)$ consisting of all homomor-

Received August 1, 1958.

* The author is extremely indebted to Professor K. Chandrasekharan for his constant advice and encouragement. The author is also thankful to Mr. N. Ramabhadran for his helpful suggestions in the exposition of the text.